Kepler's Laws

From Mars to Supermassive Black Holes

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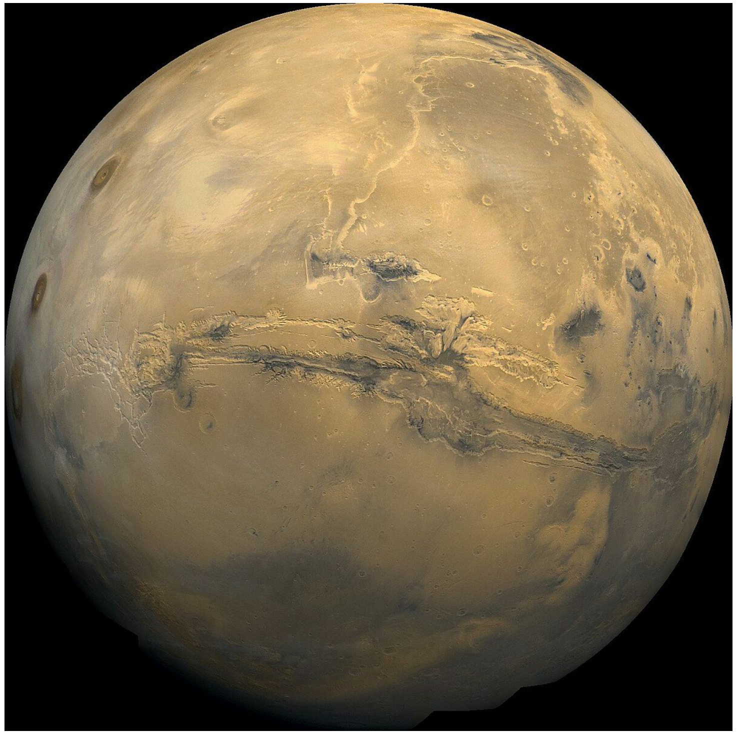
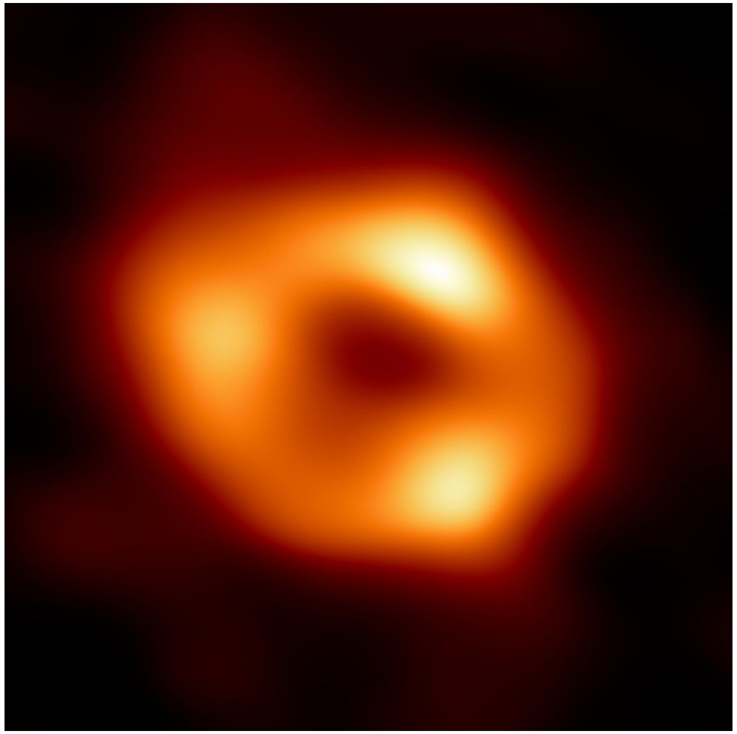
First Balkan student Summer School on Astronomy and Astrophysics
„A child from the Balkans counts the stars"
Thessaloniki, Greece, 26 – 30 August 2024

What do they have in common?



Mosaic of 102 Viking 1 Orbiter images of Mars taken on orbit 1,334, 22 February 1980

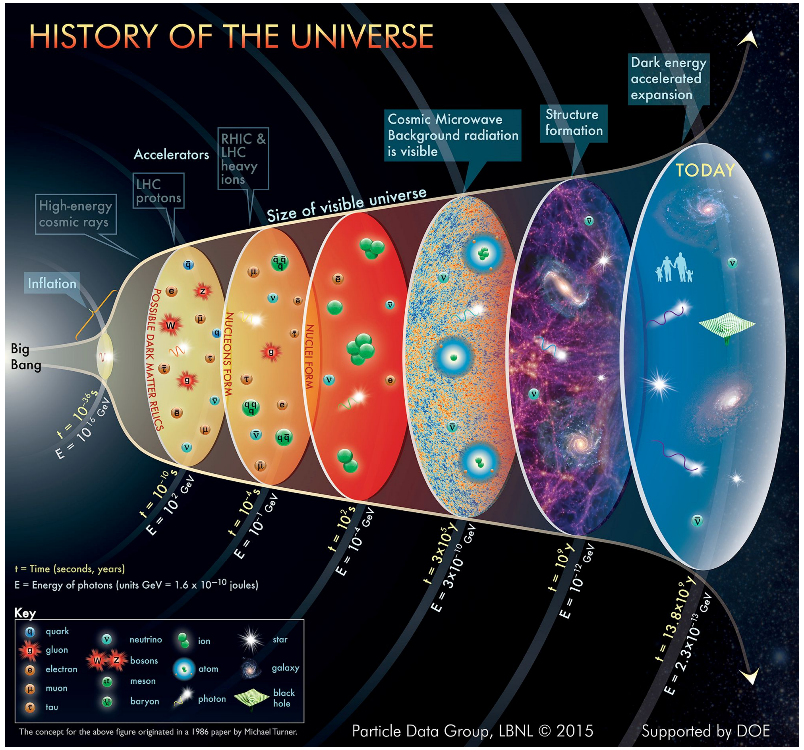
Credit: NASA / USGS



The first image of Sgr A*, the supermassive black hole at the center of our galaxy

Credit: EHT Collaboration

A brief history of astronomy



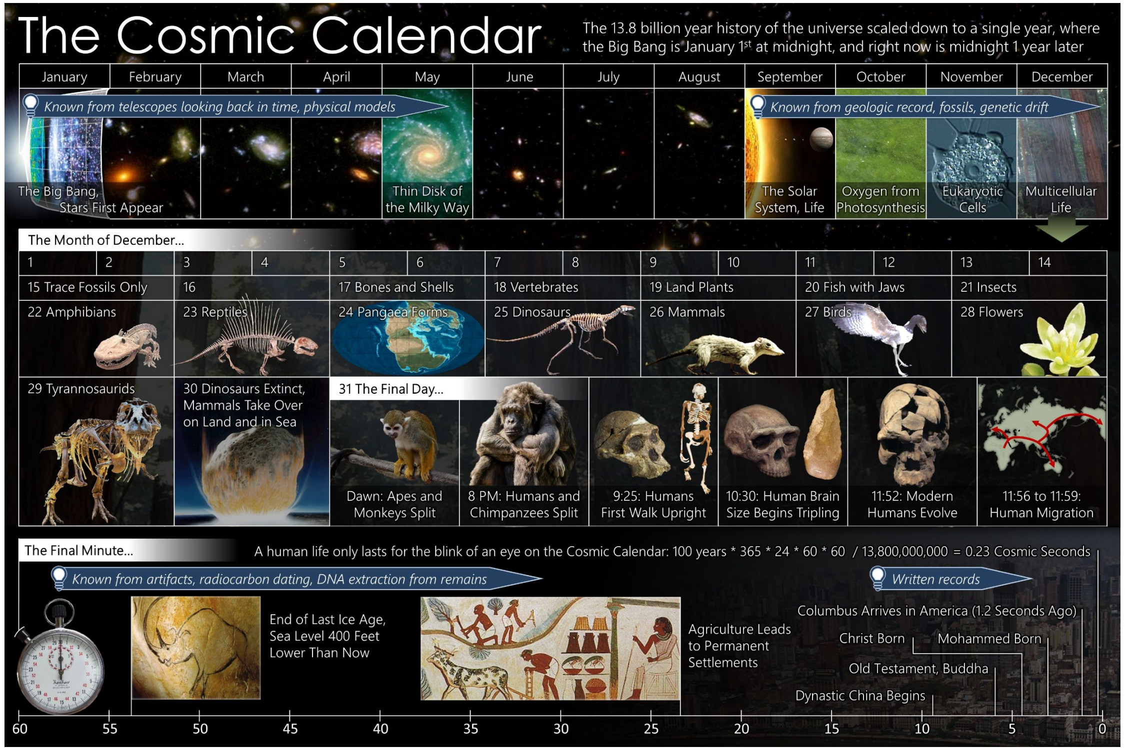
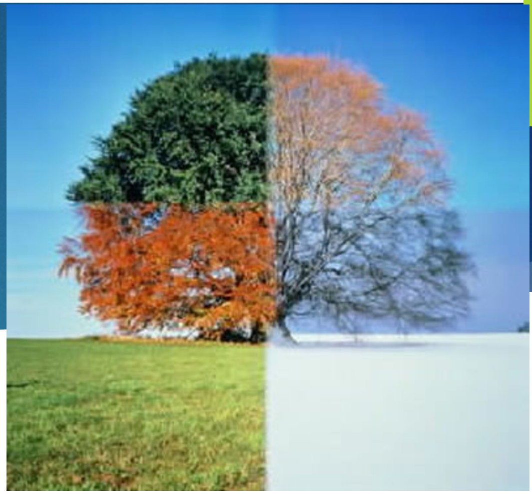


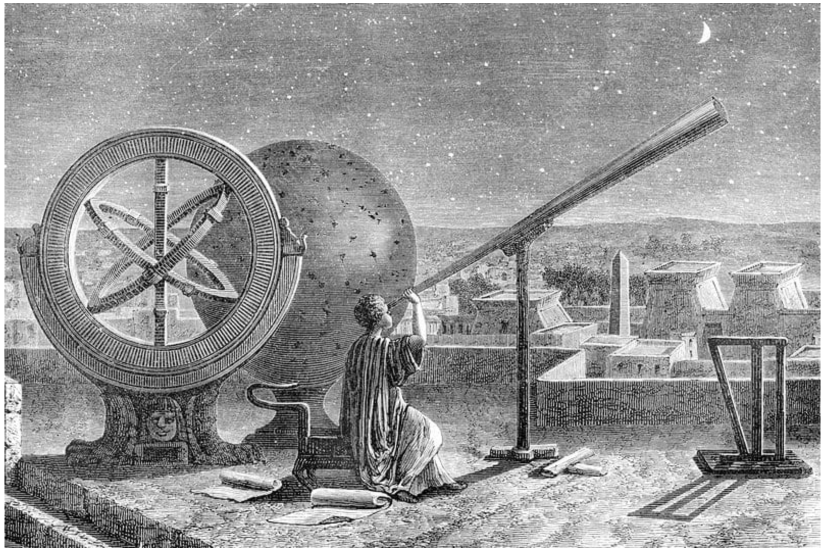
Image: Particle Data Group, LBNL

Image: Eric Fisk / Wikipedia

Why astronomy?



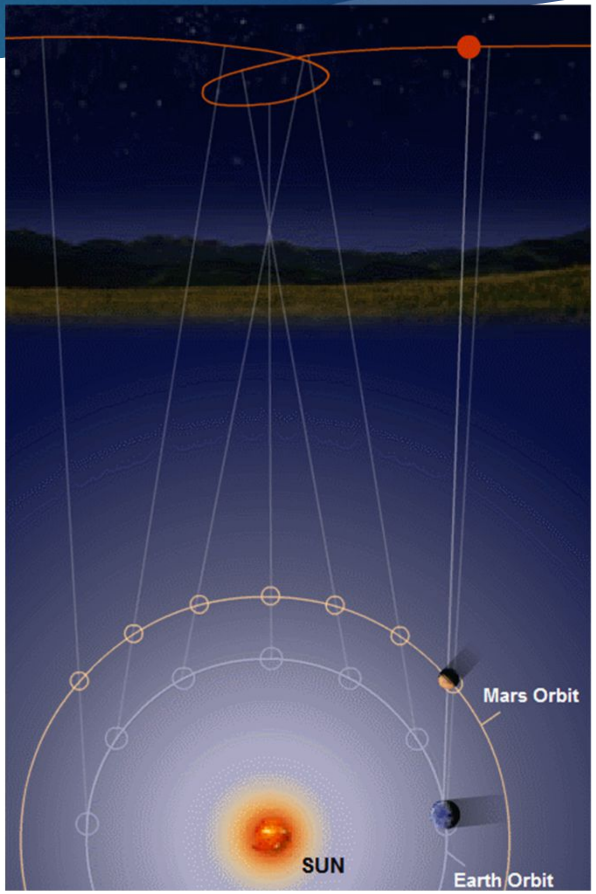
- The connection between celestial cycles and life cycles
- There was a need to predict events on Earth based on phenomena in the sky.
- Astronomy began to develop within the framework of astrology, but today, it completely rejects it as a science



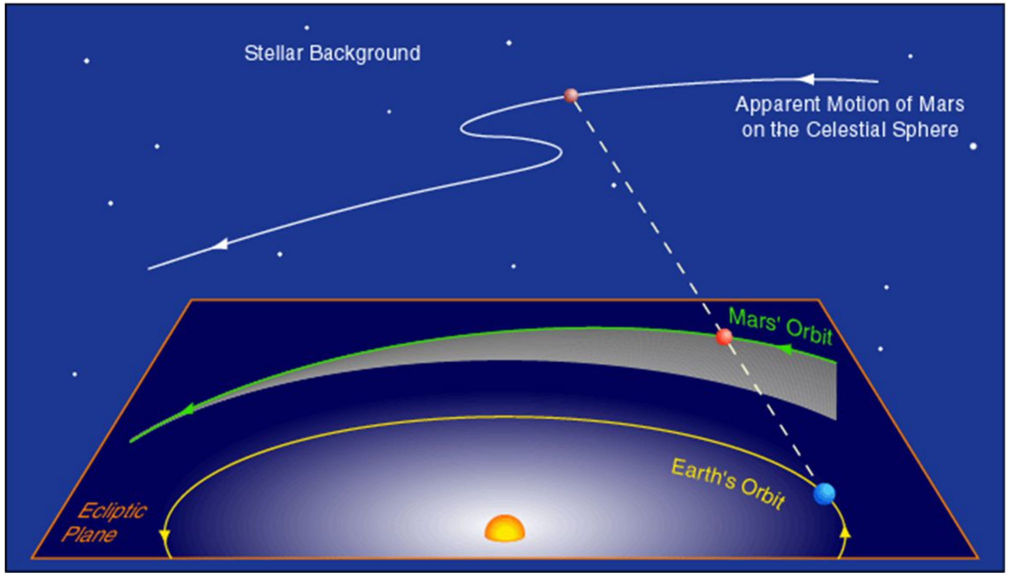
An engraving of the Greek astronomer Hipparchus (c.190-120 BC) from Vies des Savants Illustres (1877).
Source: M. Lopez, *The Epicycles of Ancient Astronomy*

Everything started with the planets

- Movement of the celestial sphere – it was long believed that the Earth is the centre of the universe
- Numerous models of the geocentric system
- The word "planet" – Greek, *Πλανήτης* (which means "wanderer")
  - Reason: annual movement across the sky and creating "loops."

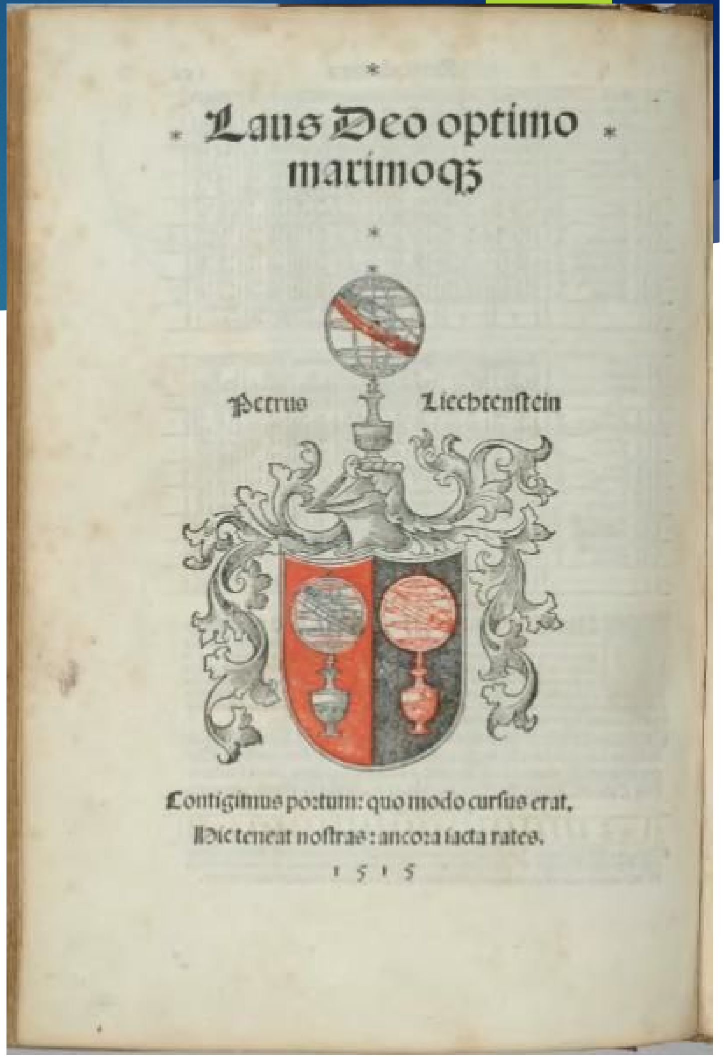


Apparent movement of planets



Geocentric model

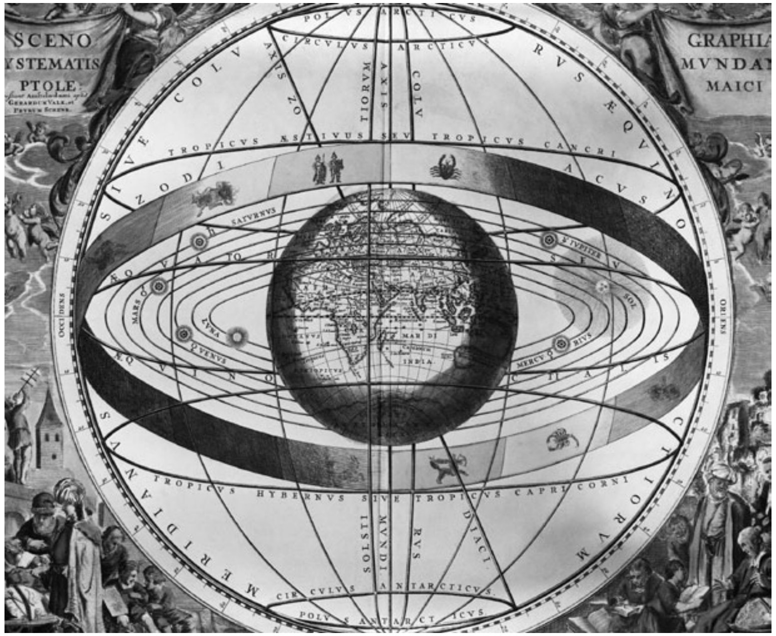
- **Aristotle** (384-382) and **Ptolemy** (90-168)
  - „*Almagest*“ - a 2nd-century mathematical and astronomical book on the apparent motions of the stars and planetary paths
- Earth – spherical in shape, at the center. Surrounding it are spheres, one for each planet, the Sun, the Moon, and the last one for the fixed stars.
- Stars are attached to the celestial sphere, with the Earth at its center, while the Sun and planets (so-called wanderers) revolve around the Earth.



Almagest, 1515 in latin, Petrus Lichtenstein (publisher)

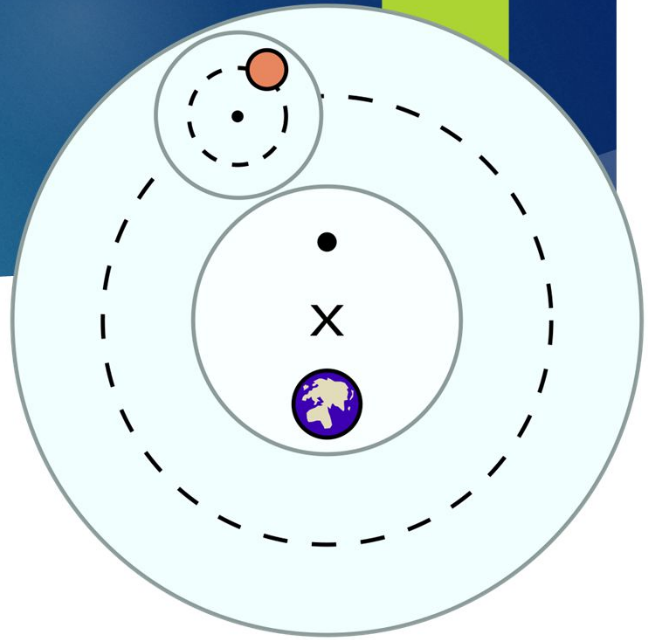
Ptolemy's model

- A geocentric universe
- accepted Aristotle's idea that the Sun and the planets revolve around a spherical Earth
- developed this idea through observation and in mathematical detail
  - he rejected the hypothesis of Aristarchus of Samos, that the Earth revolves around the Sun, he couldn't produce any evidence
- Based on observations with naked eye,
- the Universe is a set of nested, transparent spheres, Earth is in the center.
- the Moon, Mercury, Venus, and the Sun all revolved around Earth.
- Beyond the Sun, were Mars, Jupiter and Saturn, the only other planets visible to the naked eye
- Beyond Saturn was a final sphere - with all the stars fixed to it - that revolved around the other spheres.

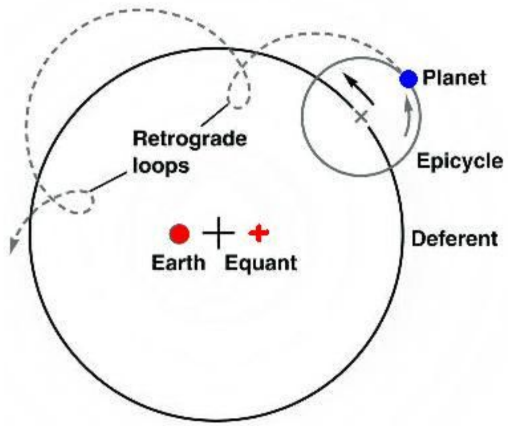


Map of the Universe according to Ptolemy, from a 17th-century Dutch atlas by Gerard Valck © Bettmann/CORBIS

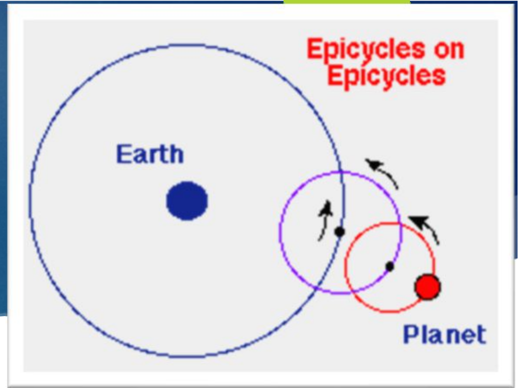
Ptolemy's model – what's new?



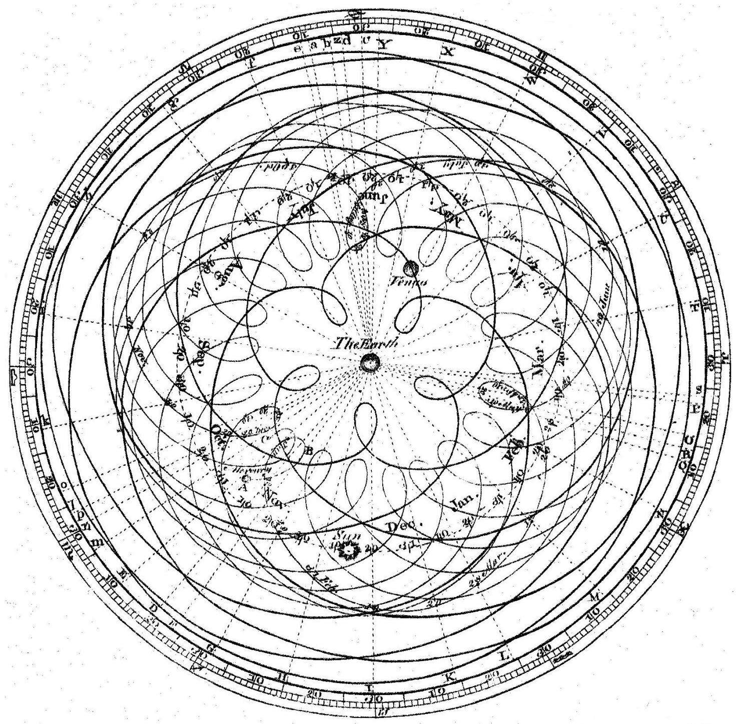
- Bodies move around the Earth in large circles called **deferents**.
- The movement of planets in "loops" involves smaller circles called **epicycles**.
- The centers of the epicycles move directly along the deferents.
- The essential elements (top right image):
  - a planet rotating on an epicycle around a deferent inside a crystalline sphere.
  - The **system's center** is marked with an X, and the Earth is slightly off the center.
  - Opposite the earth is the **equant point**, which is what the planetary deferent would rotate around.



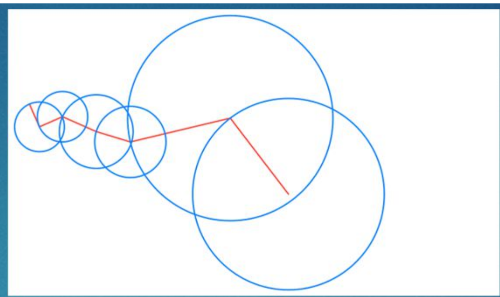
Ptolemy’s model – what’s new?

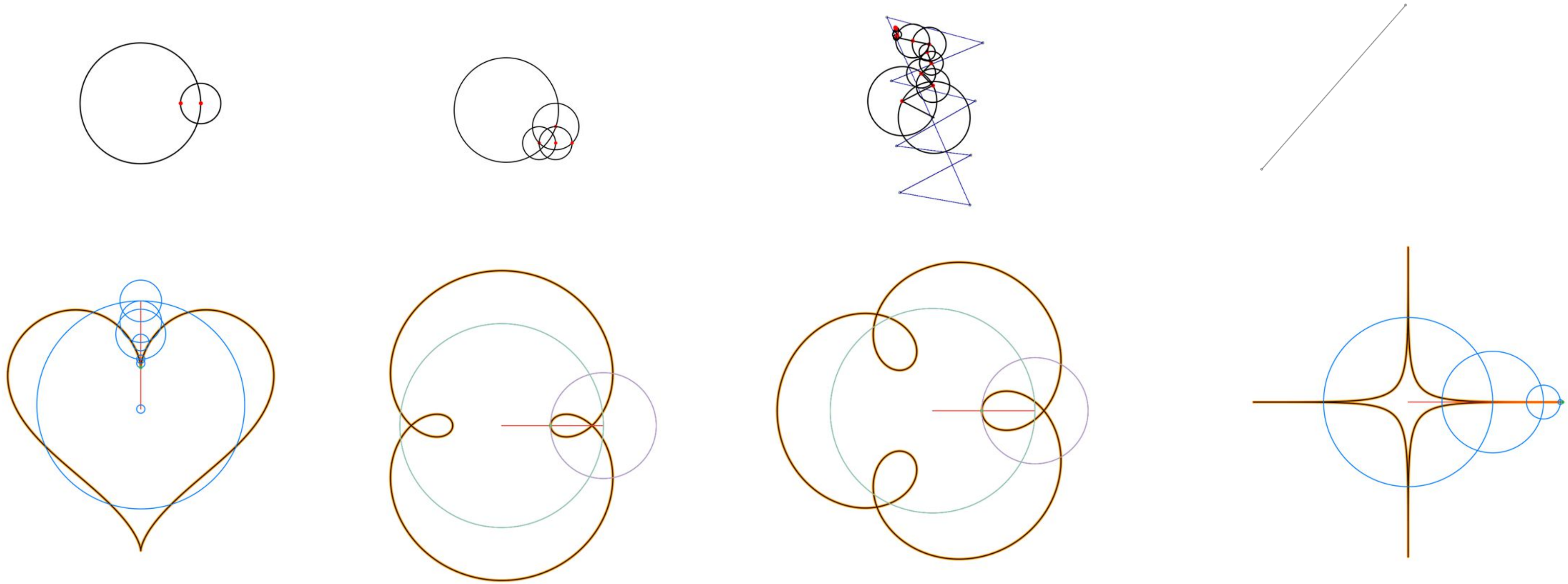


- Ptolemy's calculations of the "loops" did not match the observations.
- Corrections to the model – introducing new epicycles, epicycles of epicycles, etc.
  - Or, we can say „degrees of freedom“? ☺
- After 14 centuries – the number of epicycles grew to 80.
- The model was complex but did not correspond to the accurate picture of the sky.



However...

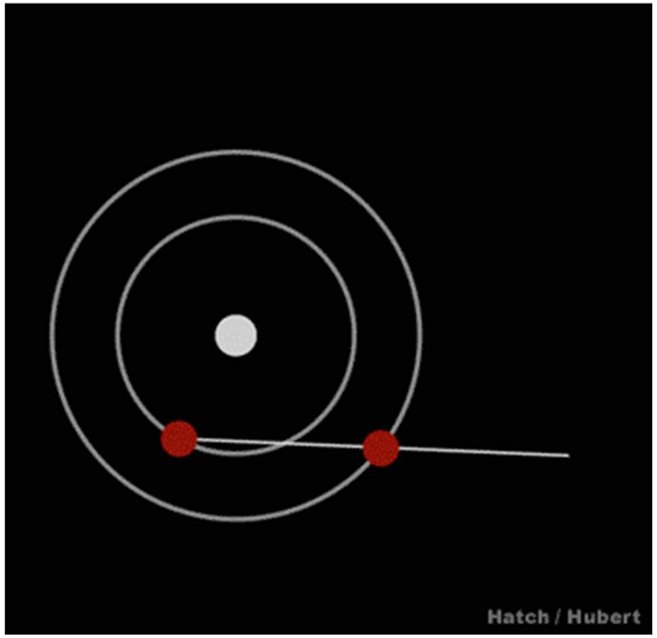


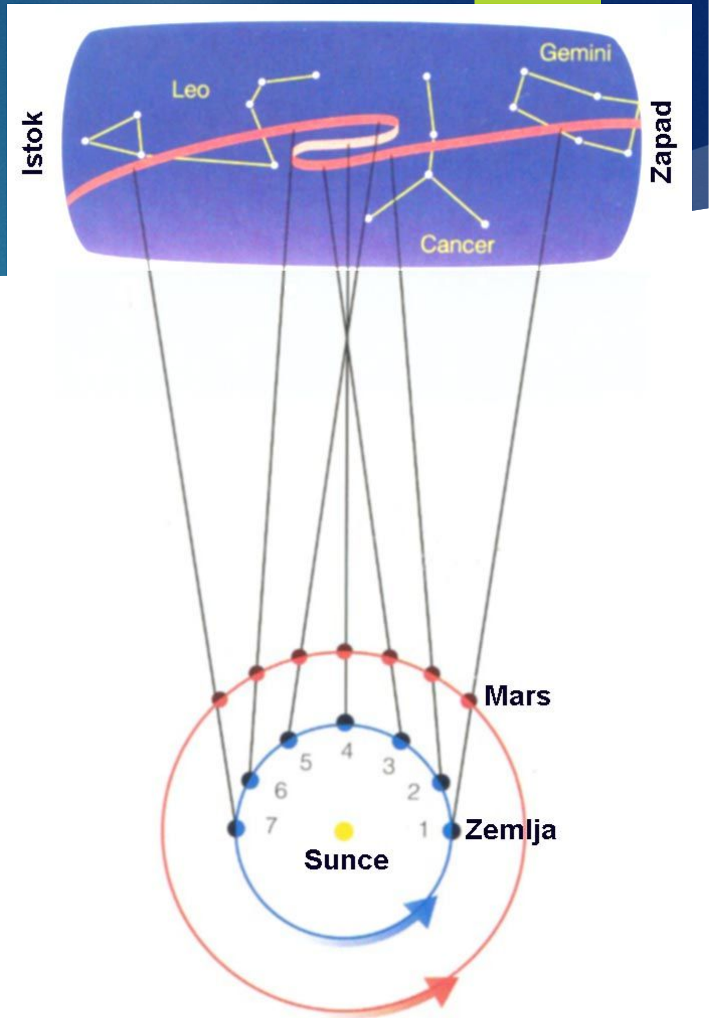


- M. Lopez, *The Epicycles of Ancient Astronomy*
- *Epicycles - Experiment with complex Fourier series*

Let's go back to the future…

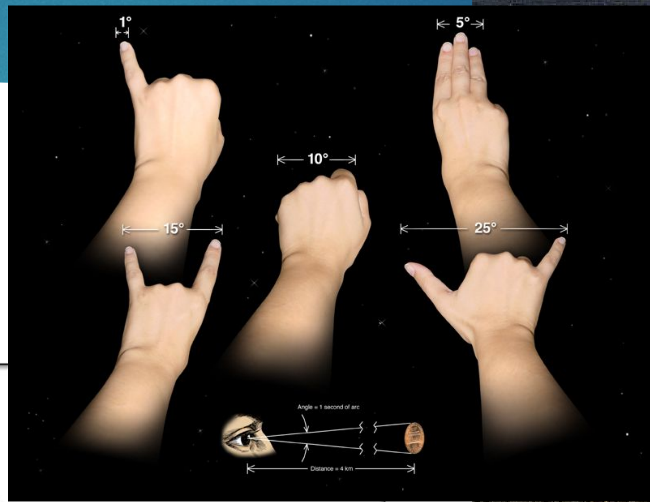
- Heliocentric system



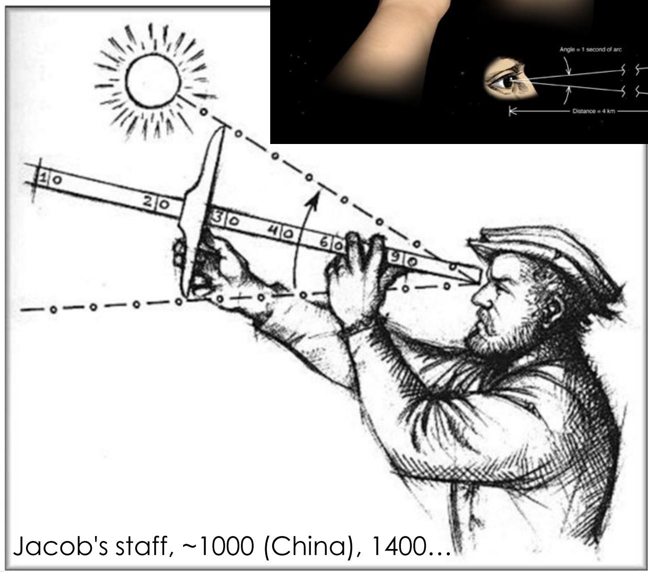


Nice idea, but how to prove it?

Sextant, 1760 and 1800+…



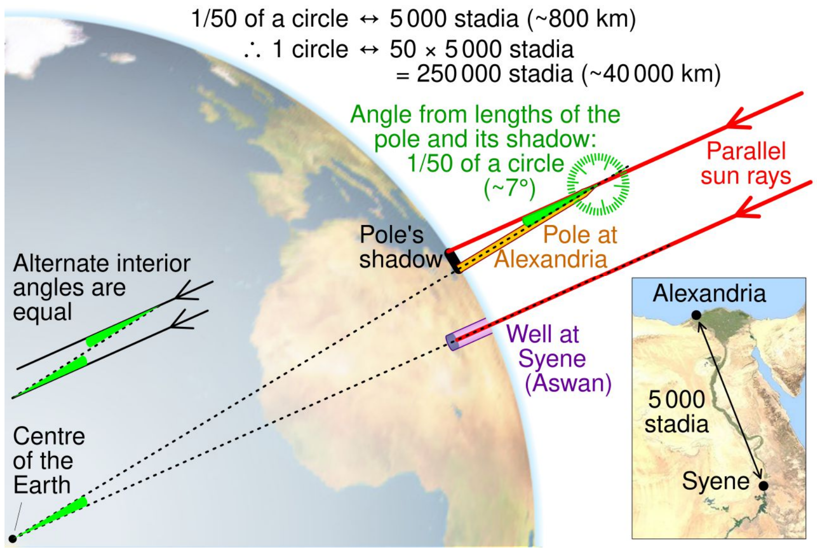
Quadrant, 1450 (possibly also the year 1200).



Jacob's staff, ~1000 (China), 1400…

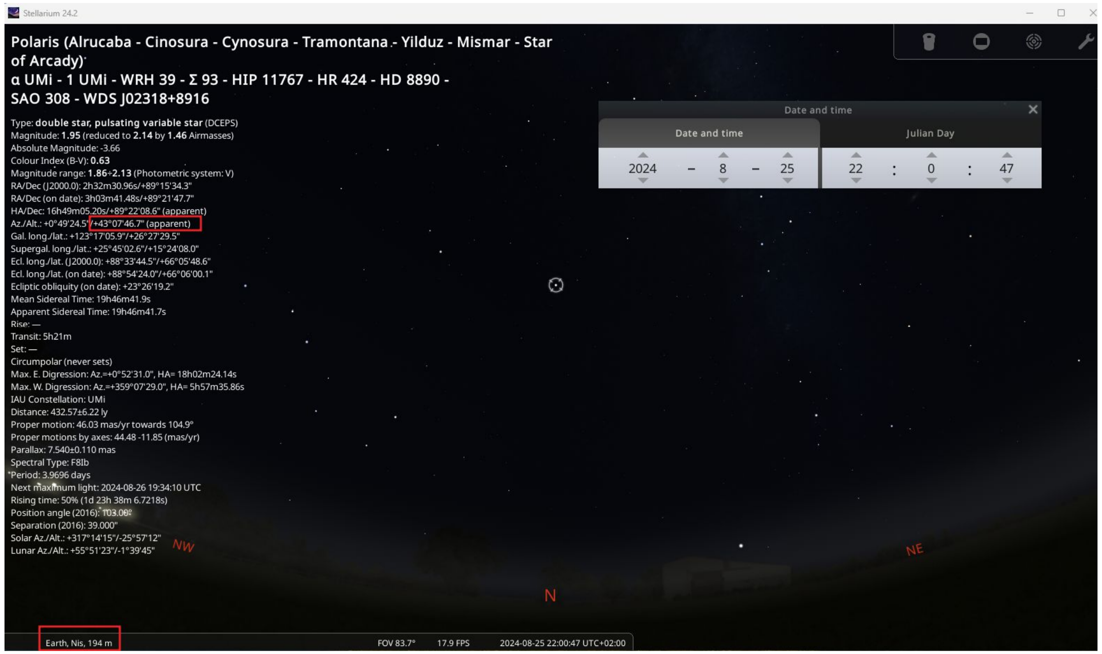
What can we do only with angles?

- Can we measure the size of the Earth?



Source: Wikimedia Common

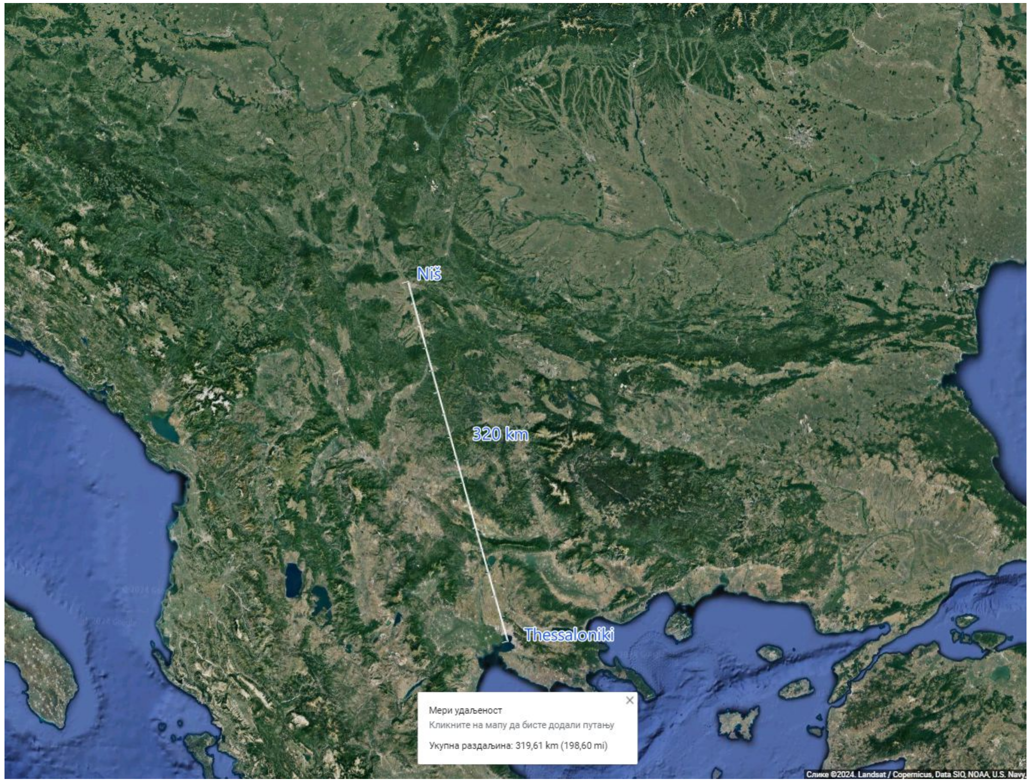
Something more astronomical?



Traveling...

- Angles of Polaris
  - Niš: 43° 07’ 46.7“
  - Thessaloniki: 40° 19’ 15.6“
  - **Difference**: $\Delta\phi = 43.13 - 40.32 = \mathbf{2.81}$

- $s = 320 \text{ km}$
- $L_{Earth} = 2\pi R = \frac{360^o}{\Delta\phi} \cdot s \approx 128.1 \cdot 320 = 40{,}996 \text{ km}$
- $\boldsymbol{R \approx 6,528 \text{ km}}$
- $s_{road} = 410 \text{ km}$ $L_{road} = 52{,}521 \text{ km}$ $R_{road} = 8{,}363 \text{ km}$
- $R_{real} = 6{,}378 \text{ km}$



Tycho Brahe (1546 - 1601)

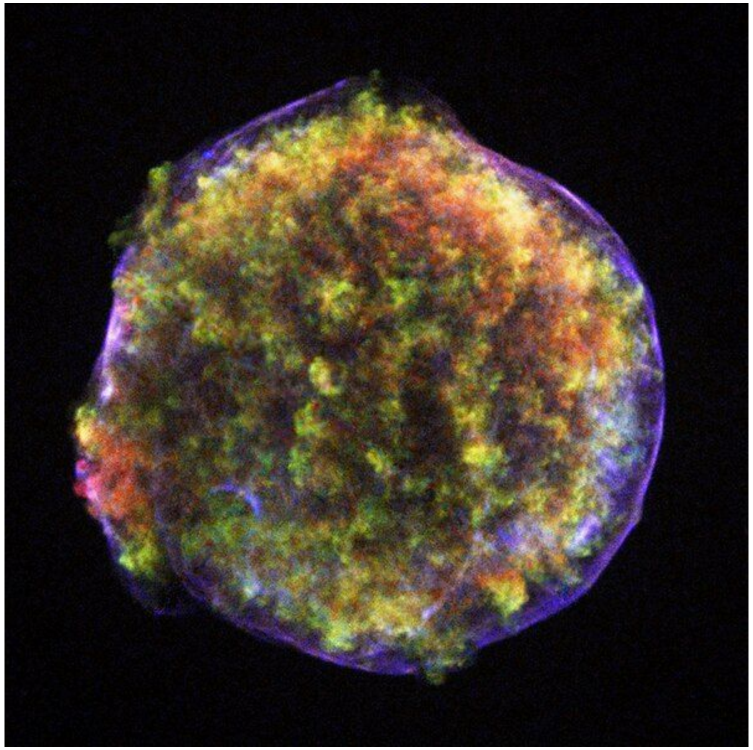
- The last significant astronomer to work without the aid of a telescope.
- He was known during his lifetime as an astronomer, astrologer, and alchemist.
- At the tender age of 14 (21 August 1560), he was captivated by the sight of a solar eclipse, a moment that sparked his lifelong passion for astronomy.
  - was greatly impressed by the fact that it had been predicted, although the prediction based on current observational data was a day off.
  - He realized that more accurate observations would be the key to making more exact predictions
- At 19, he lost part of his nose in a duel over a mathematical disagreement. He wore a prosthetic nose for the rest of his life.
  - made of brass[1] (an alloy of copper and zinc) and kept in place with wheatpaste or glue
  - Silver and gold were used only for special occasions

Tycho Brahe (oil on canvas, 1596)

[1]Danish and Czech researchers, November 2012

Tycho Brahe (1546 - 1601)

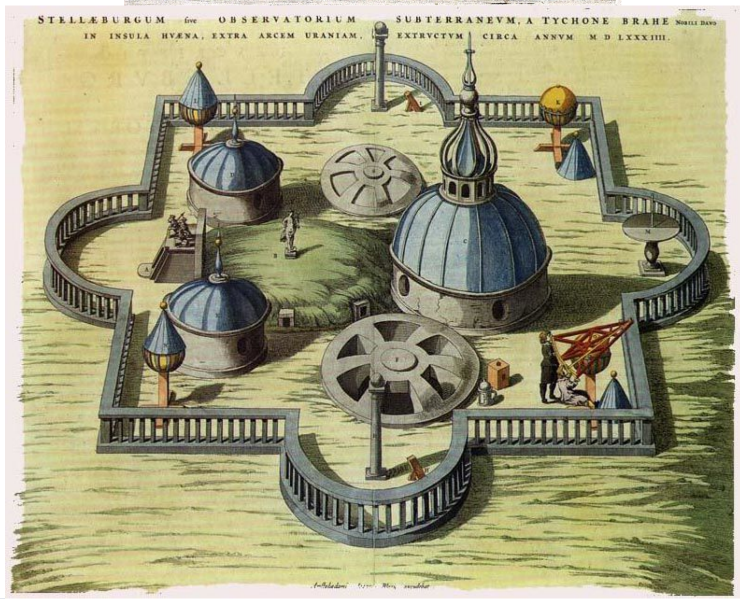
- On 11 November 1572, he observed, a very bright star, now numbered SN 1572, which had unexpectedly appeared in the constellation Cassiopeia
  - **Ancient Belief:** The celestial realm beyond the Moon is considered unchangeable (Aristotelian view).
  - **Tycho's Observation:** No daily parallax against fixed stars; object farther than the Moon and planets.
  - **Key Finding:** Object did not move relative to fixed stars; not a planet, likely a distant star.
  - ***De nova stella*** (1573): Tycho coined "*nova*" for the new star, now known as a supernova (7,500 light-years away).
  - **Impact:** Discovery pivotal in choosing astronomy as a Tycho's profession; Tycho became well-known in Europe.
  - **Criticism:** Tycho criticized those dismissing the significance of the discovery.



Tycho's Supernova Remnant (NASA/CXC/Rutgers/J.Warren & J.Hughes et al.)

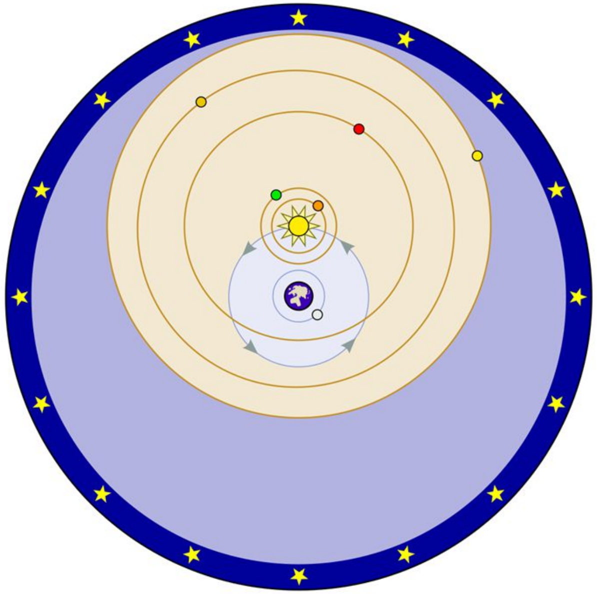
Tycho Brahe (1546 - 1601)

- **Herrevad Abbey:** With uncle Steen Bille's support, built an observatory and alchemical lab; assisted by sister Sophie Brahe.
- **King's Support:** Acknowledged by King Frederick II, who proposed building an observatory on the island of Hven.
- **Uraniborg (1576)**: Built the earliest large observatory in Christian Europe; strategically located for seclusion and focused research.
- **Pre-Telescope Era:** Tycho observed planets, moon, stars with the naked eye, compiling extensive and accurate stellar data.
- **Uraniborg's Impact:** Allowed Tycho to develop accurate solar system models and lay the groundwork for future astronomers.
- **1597:** Left Hven after disagreement with King Christian IV; **moved to Prague** and appointed imperial mathematician by Emperor Rudolf II.
- **Legacy:** Uraniborg remained a significant landmark in the history of astronomy.



Tycho Brahe (1546 - 1601)

- He was the first to teach Copernican theory in Denmark.
- However, he could not align Copernican theory with Aristotelian physics.
- He pointed out inaccuracies in Copernicus' observational data.
- He proposed a model where the Sun and Moon orbit Earth; other planets orbit the Sun - a **geo-heliocentric System**.
- Combined observational and computational benefits of Copernicus' system.
- Offered an alternative for astronomers hesitant to embrace heliocentrism.

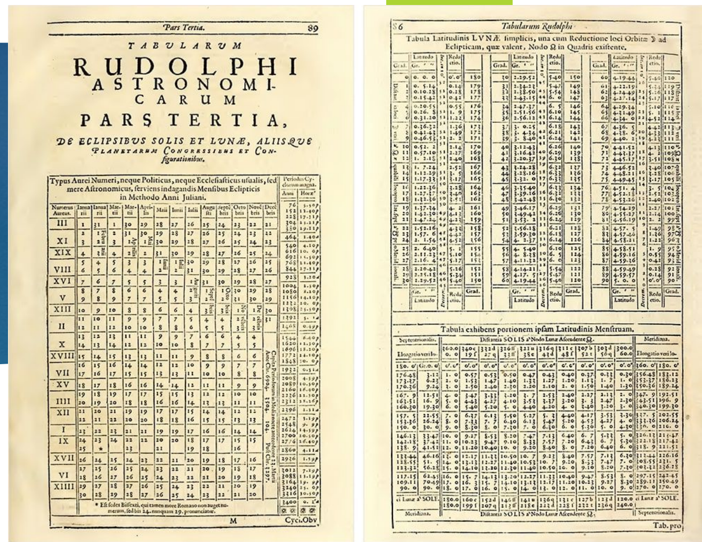


Johannes Kepler (1571 - 1630)

- German astronomer, mathematician, astrologer, natural philosopher, and writer of music
- Key figure in the 17th-century Scientific Revolution, best known for his laws of planetary motion and **his books *Astronomia nova*, *Harmonice Mundi*, and *Epitome Astronomiae Copernicanae***
- Influencing, among others **Isaac Newton**, providing one of the foundations for his theory of universal gravitation.
- One of the founders and fathers of modern astronomy, the scientific method, natural and modern science
- He was **a mathematics teacher** at a seminary school in Graz. Later, he became **an assistant to the astronomer Tycho Brahe** in Prague
- Kepler lived in an era when **the scientific landscape was complex, with no clear distinction between astronomy and astrology.** However, there was a **strong division between astronomy and physics.**

Portrait by August Köhler, 1910, after 1627 original

Johannes Kepler (1571 - 1630)



Two pages from Kepler's Rudolphine Tables showing eclipses of the Sun and Moon (September 1627)

- Kepler met Tycho Brahe at Benátky nad Jizerou (35 km from Prague), where Tycho's new observatory was built (Feb 1600).
- Kepler analyzed Tycho's Mars observations;
  - He stayed as Tycho's guest for two months, analyzing Tycho's observations of Mars;
- Tycho guarded his data closely. However, he was impressed by Kepler's theoretical ideas and granted access to more data.
- Negotiations for formal employment initially failed, leading to a brief departure.
- Kepler and Tycho reconciled and agreed on salary and living arrangements.
- Kepler returned to Graz to collect his family (June 1600) and several months later moved to Prague
- Through most of 1601, Kepler was supported directly by Tycho, who assigned him to analyze planetary observations.
- Two days after Tycho's unexpected death (24 October 1601), Kepler was appointed as his successor as the imperial mathematician and was responsible for completing his unfinished work.
- The next 11 years as an imperial mathematician would be the most productive of his life

Kepler's laws of planetary motion

- Published between 1609 and 1619; the laws describe the orbits of planets around the Sun.
- They modified the heliocentric theory of Nicolaus Copernicus, replacing its circular orbits and epicycles with elliptical trajectories, and explaining how planetary velocities vary

Copernicus	Kepler's law
The planetary orbit is a circle with epicycles	The planetary orbit is not a circle with epicycles, but an ellipse.
The Sun is approximately at the center of the orbit.	The Sun is not at the center but at a focal point of the elliptical orbit.
The speed of the planet in the main orbit is constant.	Neither the linear speed nor the angular speed of the planet in the orbit is constant, but the area speed is constant

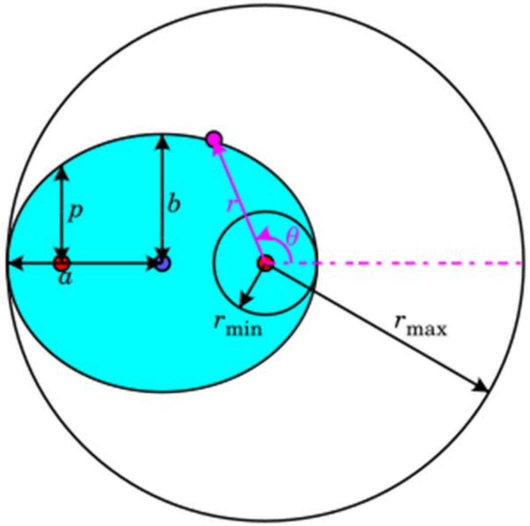
The **First law**

- **The orbit of every planet is an ellipse with the Sun at one of the two focuses.**
- Equation of ellipse $r = \frac{p}{1+\varepsilon\cos\theta}$; eccentricity $\varepsilon = \sqrt{1 - \frac{b^2}{a^2}}$
  - where p is the orbital parameter, ε is the eccentricity of the ellipse, r is the distance from the Sun to the planet, and θ is the angle to the planet's current position from its closest approach, as seen from the Sun
  - For $\theta = 90^o$ and $\theta = 270^o$, we have $r = p$.
- Arithmetic, geometric and harmonic mean:

$$a = \frac{r_{max} + r_{min}}{2} = \frac{p}{1 - \epsilon^2}$$

$$b = \sqrt{r_{max} \cdot r_{min}} = \frac{p}{\sqrt{1 - \epsilon^2}}$$

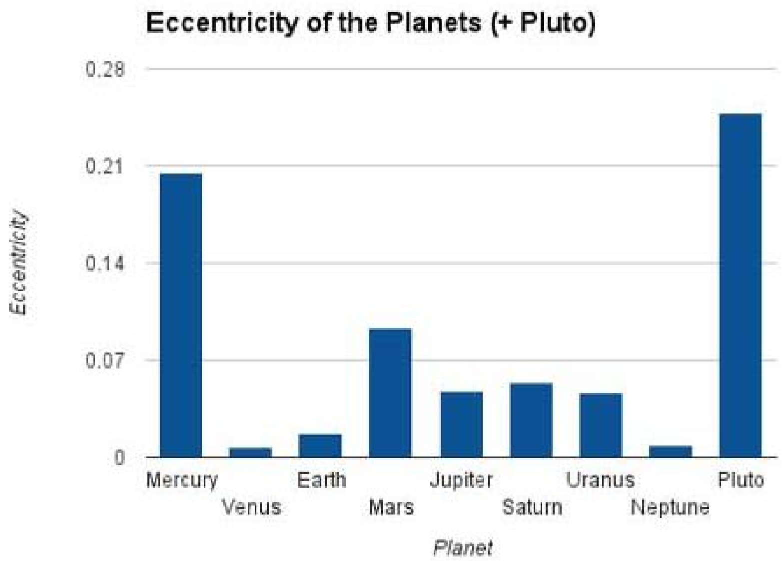
$$p = \left(\frac{r_{min}^{-1} + r_{max}^{-1}}{2}\right)^{-1}$$

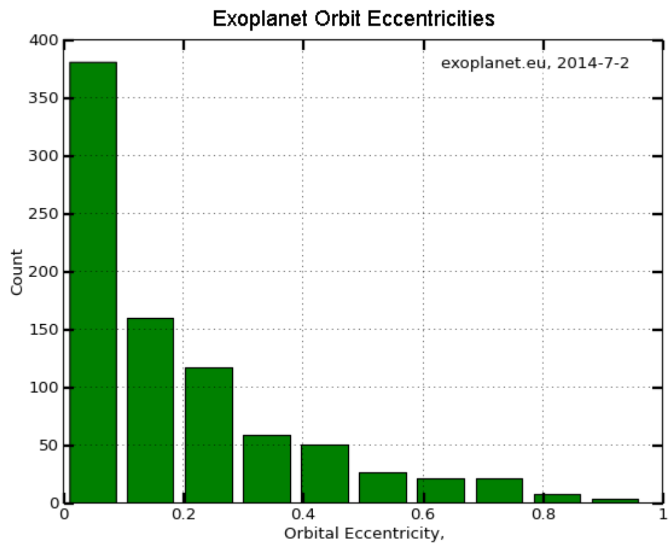


Heliocentric (polar) coordinate system (r, θ) for ellipse. The image shows: semi-major axis a, semi-minor axis b and orbital parameter (semi-latus rectum) p; center of ellipse and its two focuses marked by large dots. For $\theta = 0°$, $r = r_{min}$ and for $\theta = 180°$, $r = r_{max}$.

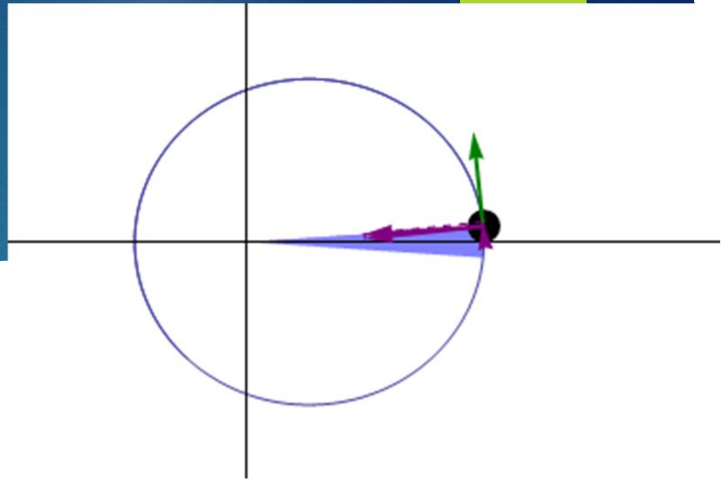
The **First law**

- The orbits of the planets have small eccentricities (they are not much different from circular), except in the case of Mercury and the "former" planet Pluto.
- The orbits of planetary satellites, asteroids, and periodic comets are also elliptical.





The **Second law**



- **A line joining a planet, and the Sun sweeps out equal areas during equal time intervals.**

$$dA = \frac{1}{2} r \cdot r d\theta$$

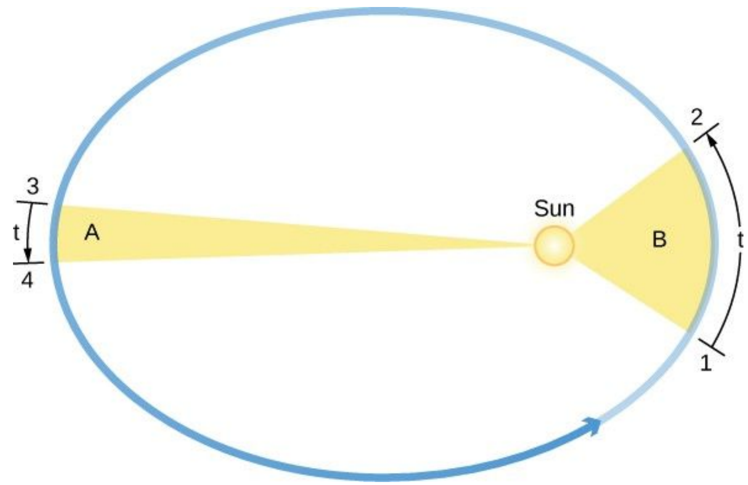
$$\frac{\boldsymbol{dA}}{\boldsymbol{dt}} = \frac{\boldsymbol{r^2}}{\boldsymbol{2}} \frac{\boldsymbol{d\theta}}{\boldsymbol{dt}}$$

constant areal (sector) velocity

$$T \cdot \frac{r^2}{2} \frac{d\theta}{dt} = \pi ab$$

$$n = \frac{2\pi}{T} = \frac{360^o}{T} \quad \Rightarrow \quad r^2 d\theta = ab \cdot n\, dt$$

$$\boxed{\frac{\boldsymbol{dA}}{\boldsymbol{dt}} = \frac{\boldsymbol{abn}}{\boldsymbol{2}} = \frac{\boldsymbol{\pi ab}}{\boldsymbol{n}}}$$



The **Third Law**

- **The ratio of the square of an object's orbital period (T) with the cube of the semi-major axis (a) of its orbit is the same for all objects orbiting the same primary.**

$$\frac{a_1^3}{T_1^2} = \frac{a_2^3}{T_2^2} = \frac{a_3^3}{T_3^2} = \cdots = const$$

- Using Newton's law of gravitation (1687), 3rd Kepler's law can be found in the **case of a circular orbit** by setting the centripetal force equal to the gravitational force:

$$mr\omega^2 = G\frac{mM}{r^2}$$

$$mr\left(\frac{2\pi}{T}\right)^2 = G\frac{mM}{r^2}$$

$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3 = const \cdot r^3$$

$$\frac{1}{(365.25)^2} \approx 7.5 \cdot 10^{-6}$$

$$\frac{a^3}{T^2} = \frac{G(M+m)}{4\pi^2} \approx \frac{GM}{4\pi^2} \approx 7.5 \cdot 10^{-6}\,\frac{\mathrm{AU}^3}{\mathrm{day}^2}$$

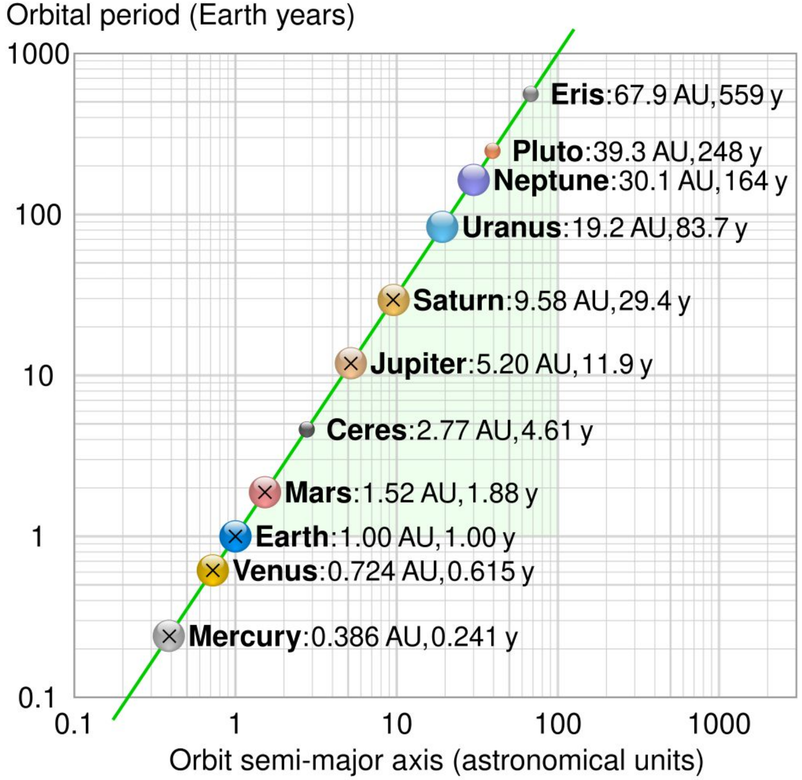
The **Third law**

Data used by Kepler (1618)

Planet	Mean distance to sun (AU)	Period (days)	$\frac{R^3}{T^2}$ (10^{-6} AU^3/day^2)
Mercury	0.389	87.77	7.64
Venus	0.724	224.70	7.52
Earth	1	365.25	7.50
Mars	1.524	686.95	7.50
Jupiter	5.2	4332.62	7.49
Saturn	9.510	10759.2	7.43

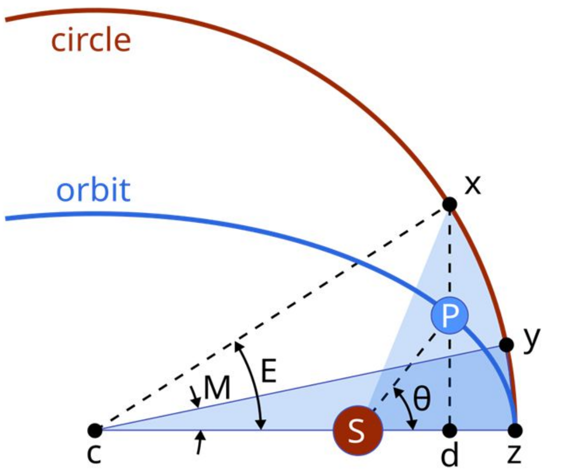
Modern data (Wolfram Alpha Knowledgebase 2018)

Planet	Semi-major axis (AU)	Period (days)	$\frac{R^3}{T^2}$ (10^{-6} AU^3/day^2)
Mercury	0.38710	87.9693	7.496
Venus	0.72333	224.7008	7.496
Earth	1	365.2564	7.496
Mars	1.52366	686.9796	7.495
Jupiter	5.20336	4332.8201	7.504
Saturn	9.53707	10775.599	7.498
Uranus	19.1913	30687.153	7.506
Neptune	30.0690	60190.03	7.504



Position of planets (1)

- The first and the second law is used to compute the position of a planet as a function of time:
  1. Compute the **mean motion** $n = (2\pi)/T$, where T is the period.
  2. Compute the **mean anomaly** $M = M_0 + n \cdot \Delta t$, and $\Delta t = t - T_0$ where t is time of calculation and T_0 is epoch; M_0 is mean time of the epoch for the planet.
  3. Compute the **eccentric anomaly** E by solving Kepler's equation
  $$M = E - \epsilon \cdot \sin E,$$
  where ϵ is the eccentricity.
  4. Compute the **true anomaly** θ by solving the equation:
  $$(1-\epsilon)\tan^2\frac{\theta}{2} = (1+\epsilon)\tan^2\frac{E}{2}$$
  5. Compute the **heliocentric distance** r:
  $$r = a(1 - \epsilon\cos E)$$
  where a is the semimajor axis.

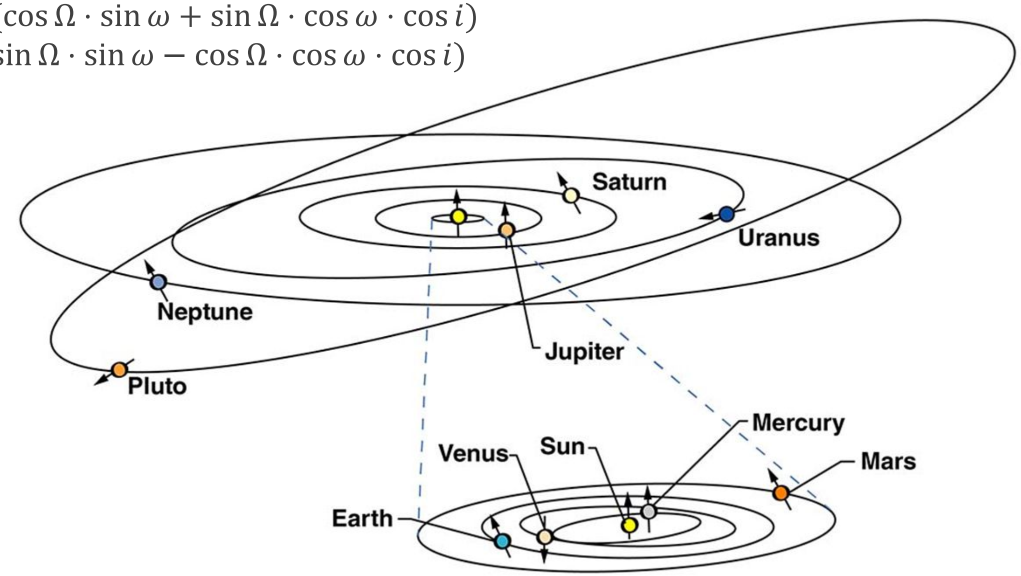


Source: CheCheDaWaff/Wikimedia Commons

Position of planets (2)

6. Compute the Heliocentric Coordinates (x', y', z')
   - In orbital plane coordinates are $x' = r \cdot \cos\theta$, $y' = r \cdot \sin\theta$, $z' = 0$
   - Convert these to heliocentric ecliptic coordinates:

$$x = x' \cdot (\cos\Omega \cdot \cos\omega - \sin\Omega \cdot \sin\omega \cdot \cos i) - y' \cdot (\cos\Omega \cdot \sin\omega + \sin\Omega \cdot \cos\omega \cdot \cos i)$$
$$y = x' \cdot (\sin\Omega \cdot \cos\omega \cdot \cos\Omega \cdot \sin\omega \cdot \cos i) + y' \cdot (\sin\Omega \cdot \sin\omega - \cos\Omega \cdot \cos\omega \cdot \cos i)$$
$$z = x' \sin\omega \cdot \sin i + y' \cdot \cos\omega \cdot \sin i$$



Let's find Saturn in the sky ☺

- Repeat steps 1 - 6 for Earth and Saturn!
- Orbital elements (Epoch J2000.0)

	Earth	Saturn
Semi-major axis (a)	1.00000261 AU	9.53707032 AU
Eccentricity (e)	0.01671123	0.05415060
Inclination (i)	0.00005°	2.485240°
Longitude of Ascending Node (Ω)	348.73936°	113.6624°
Argument of Perihelion (ω)	114.20783°	336.0139°
Mean Anomaly (M_0) at Epoch	357.51716°	320.3462°
Epoch (T_0)	J2000.0	J2000.0
	01.01.2000 @12:00:00	

Position of planets (3)

- We want to calculate the position of Saturn (**right ascension**, α, and **declination**, δ) in the sky

7. Calculate the Geocentric Position of Saturn

$$x_{geocentric} = x_{saturn} - x_{earth}$$
$$y_{geocentric} = y_{saturn} - y_{earth}$$
$$z_{geocentric} = z_{saturn} - z_{earth}$$

8. Convert to Equatorial Coordinates, $\epsilon \approx 23.44^o$

$$x_{equatorial} = x_{geocentric}$$
$$y_{equatorial} = y_{geocentric} \cdot \cos\epsilon - z_{geocentric} \cdot \sin\epsilon$$
$$z_{equatorial} = y_{geocentric} \cdot \sin\epsilon + z_{geocentric} \cdot \cos\epsilon$$

9. Calculate Right Ascension (RA) and Declination (Dec):

$$\alpha = \tan^{-1}\frac{x_{equatorial}}{y_{equatorial}} \qquad \delta = \sin^{-1}\frac{z_{equatorial}}{\sqrt{x_{equatorial}^2+y_{equatorial}^2+z_{equatorial}^2}}$$

Saturn, tonight at 23:00

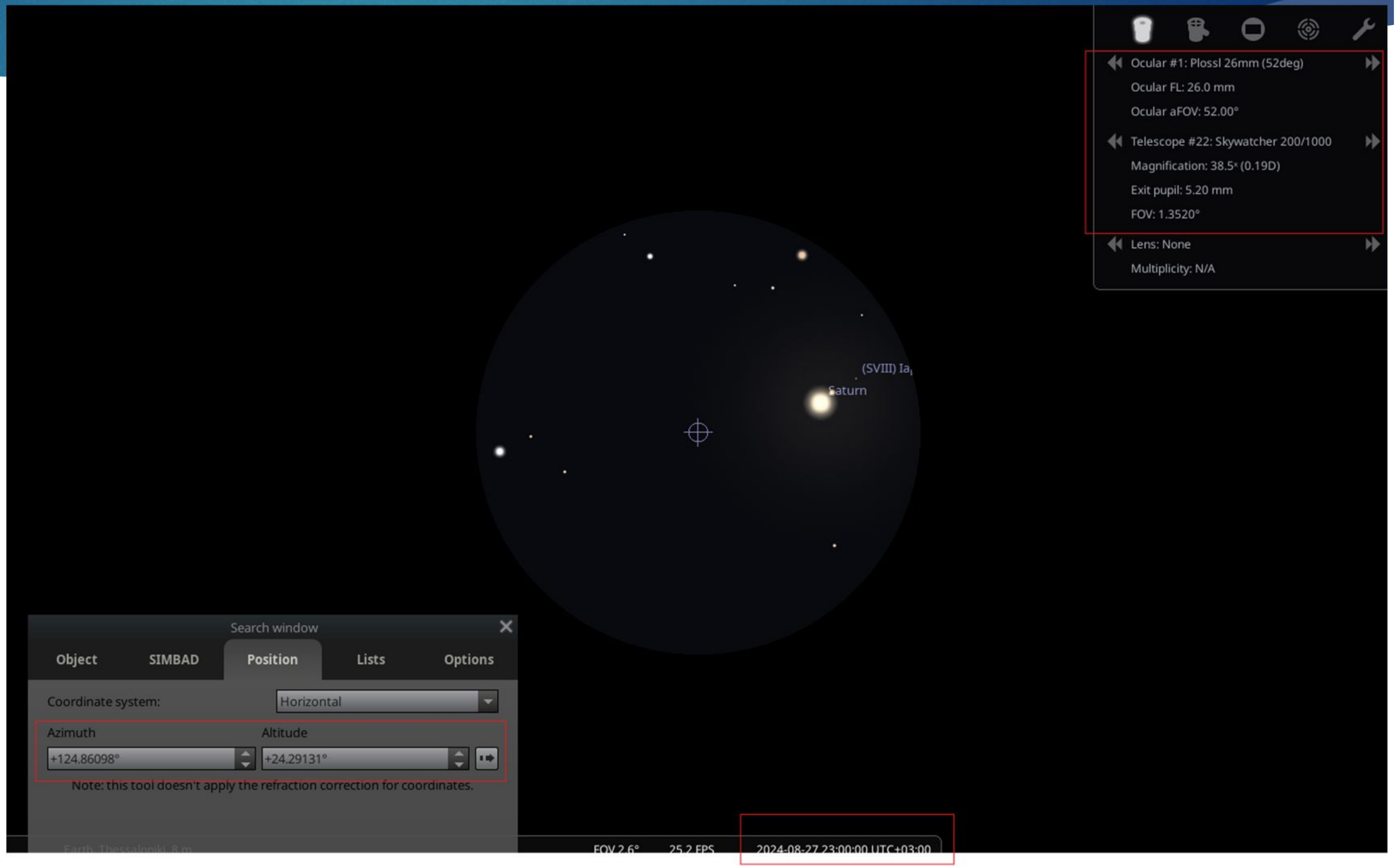
- RA (hours): 23.23
- Dec (degrees): -7.33

- Alt. (degrees): 24.29
- Az. (degrees): 124.86

Saturn, tonight at 23:00

- RA (hours): 23.23
- Dec (degrees): -7.33

- Alt. (degrees): 24.29
- Az. (degrees): 124.86

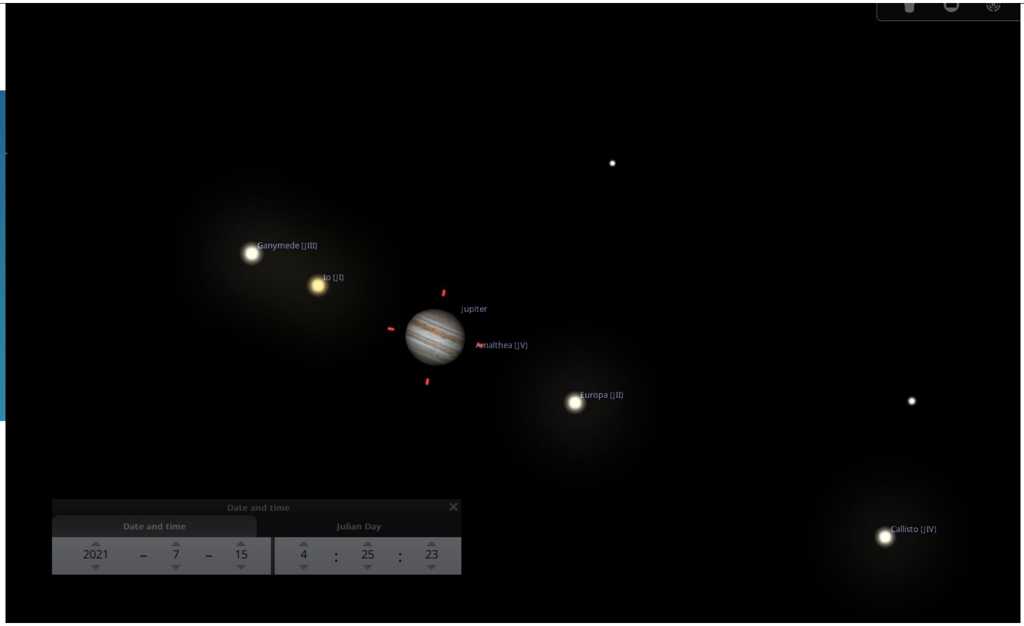


Mass of Jupiter

- We cannot directly measure everything in the universe(the masses and distances of the planets and their moons). However, we can deduce some properties of celestial bodies from their motions, although we cannot directly measure them
- The Third Kepler's law: $\frac{T^2}{a^3} = \frac{4\pi^2}{G \cdot M}$

$$M = \frac{a^3}{T^2}$$

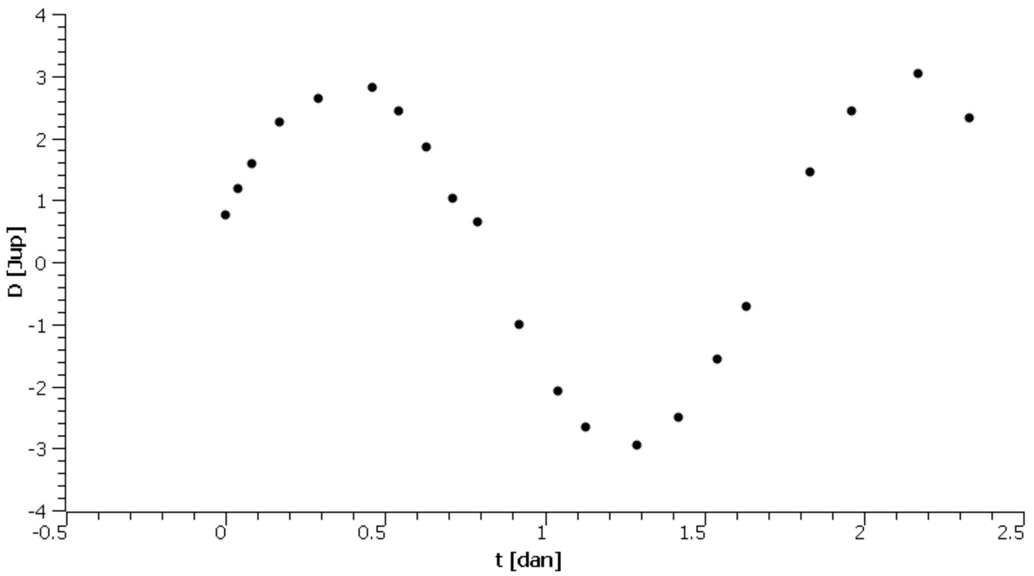
- M – mass of the central body (in solar mass),
- a – semi-major axis (in AU),
- T – orbital period (in years)



Observation (Stellarium)

time	ΔT	sig	min	sec	Jupiter	D [Jup]	D [AU]
2459409.565	0.00	1	0	35.41	46.51	0.7613	1.63E-05
2459409.607	0.04	1	0	55.26	46.51	1.1881	2.55E-05
2459409.649	0.08	1	1	13.87	46.51	1.5883	3.41E-05
2459409.732	0.17	1	1	44.86	46.51	2.2546	4.84E-05
2459409.857	0.29	1	2	2.12	46.51	2.6257	5.63E-05
2459410.024	0.46	1	2	11.16	46.51	2.8200	6.05E-05
2459410.107	0.54	1	1	53.27	46.51	2.4354	5.23E-05
2459410.191	0.63	1	1	26.67	46.51	1.8635	4.00E-05
2459410.274	0.71	1	0	47.87	46.51	1.0292	2.21E-05
2459410.357	0.79	1	0	30.00	46.51	0.6450	1.38E-05
2459410.482	0.92	-1	0	47.08	46.51	-1.0123	-2.17E-05
2459410.607	1.04	-1	1	36.94	46.51	-2.0843	-4.47E-05
2459410.691	1.13	-1	2	3.28	46.51	-2.6506	-5.69E-05
2459410.857	1.29	-1	2	16.76	46.51	-2.9404	-6.31E-05
2459410.982	1.42	-1	1	56.61	46.51	-2.5072	-5.38E-05
2459411.107	1.54	-1	1	12.66	46.51	-1.5622	-3.35E-05
2459411.191	1.63	-1	0	33.35	46.51	-0.7171	-1.54E-05
2459411.399	1.83	1	1	7.69	46.51	1.4554	3.12E-05
2459411.524	1.96	1	1	53.30	46.51	2.4360	5.23E-05
2459411.732	2.17	1	2	21.25	46.51	3.0370	6.52E-05
2459411.899	2.33	1	1	48.32	46.51	2.3290	5.00E-05

Jupiter [km]	139,820
AU [km]	149,597,871
Jupiter [AU]	46,606.67



Observation (results)

$$y = a\cos(\omega(t - c)) + b$$

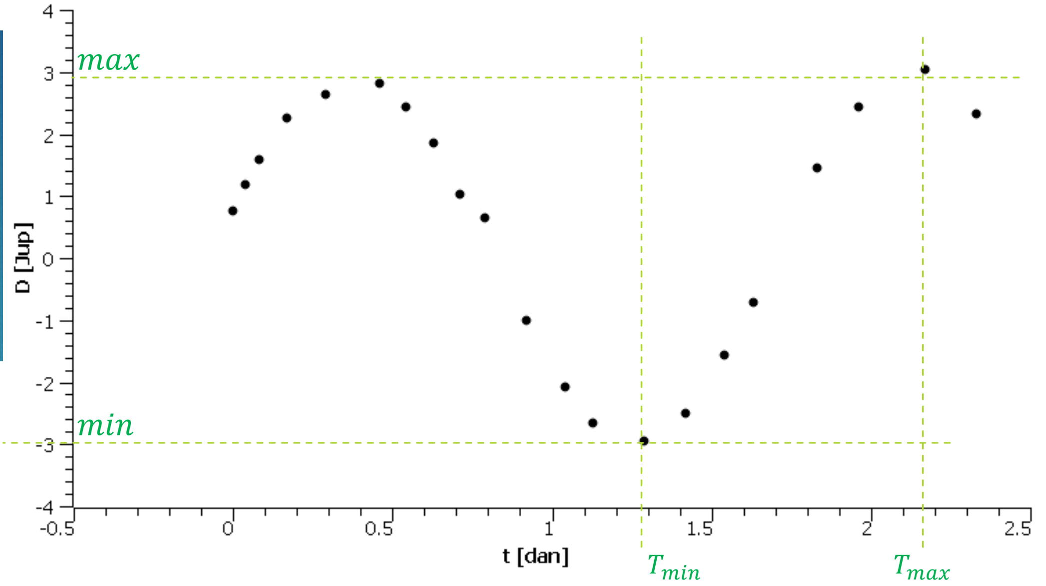
$$b = \frac{max + min}{2} = \frac{1}{2}(2.9 + (-3.05)) = \frac{-0.15}{2} = -0.075$$

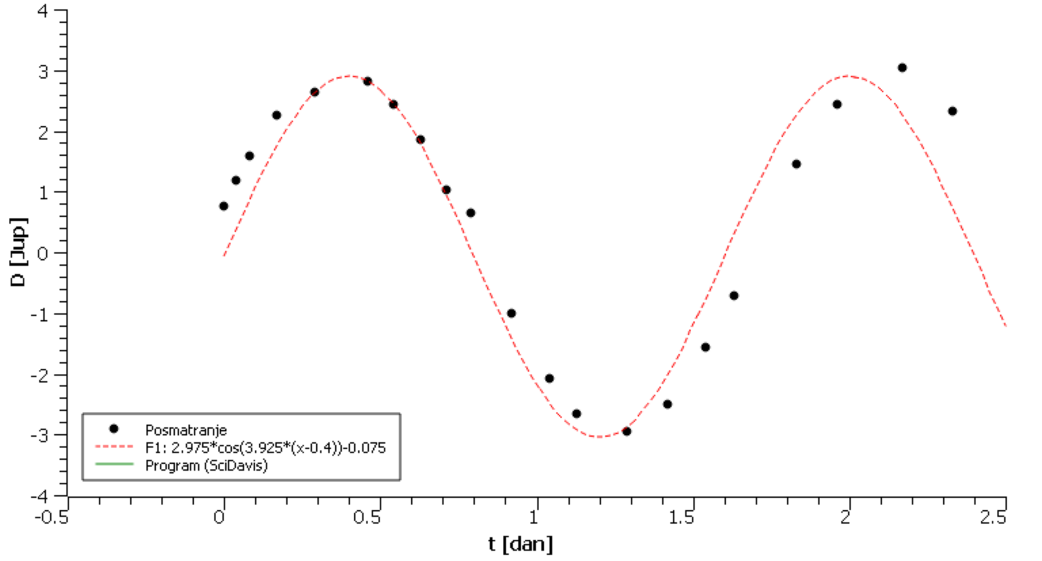
$$a = \frac{max - min}{2} = \frac{1}{2}(2.9 - (-3.05)) = \frac{5.95}{2} = 2.975$$

$$T = \frac{2\pi}{\omega} = 2(T_{max} - T_{min}) = 2(2.1 - 1.3) = 2 \cdot 0.8 = 1.6$$

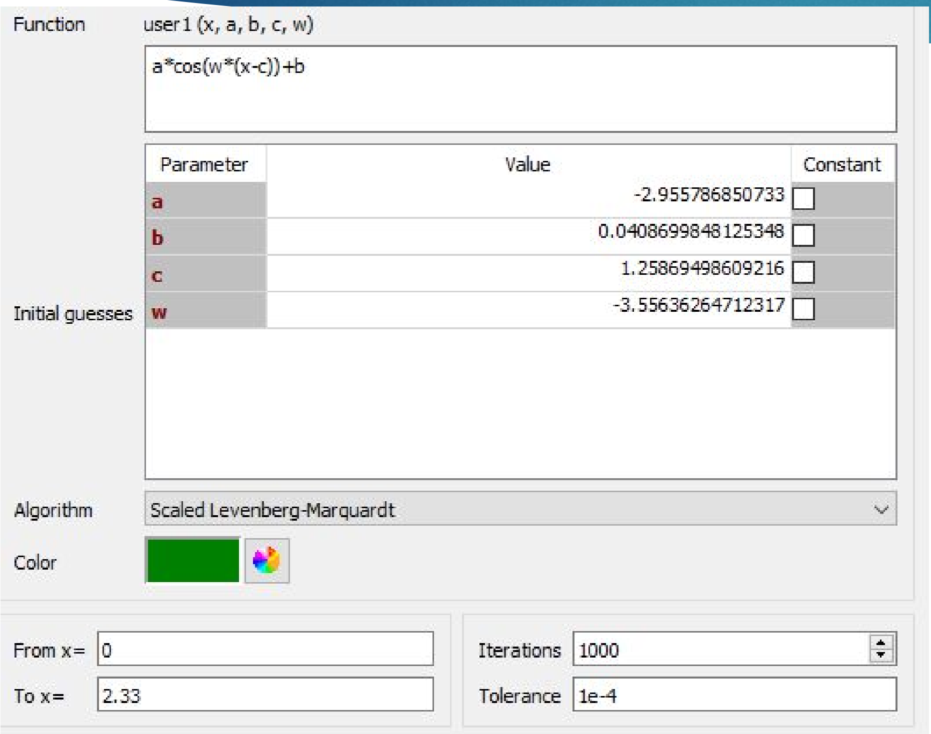
- $\omega = \frac{2\pi}{T} = 3.925$

- $c = 0.4$



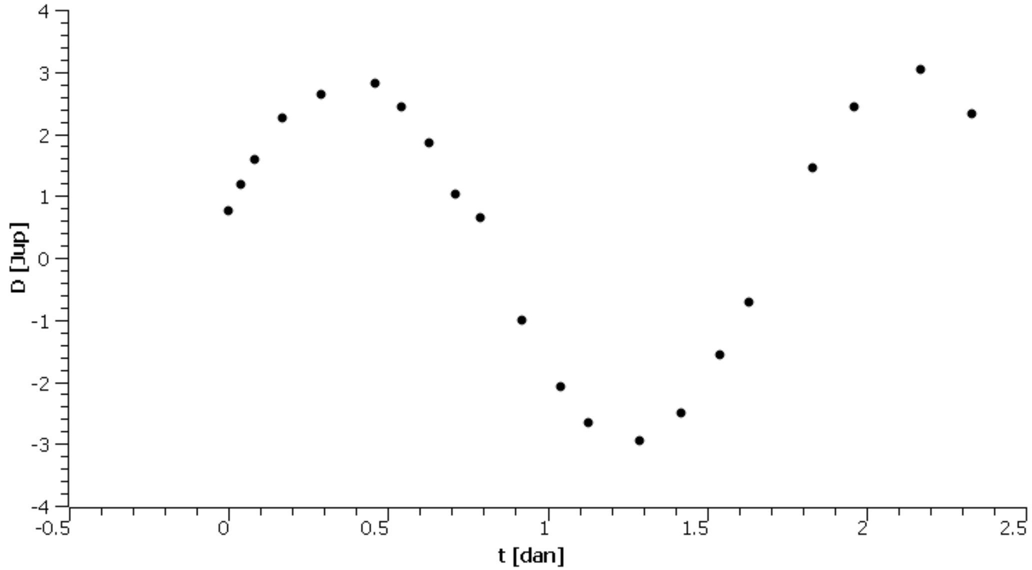


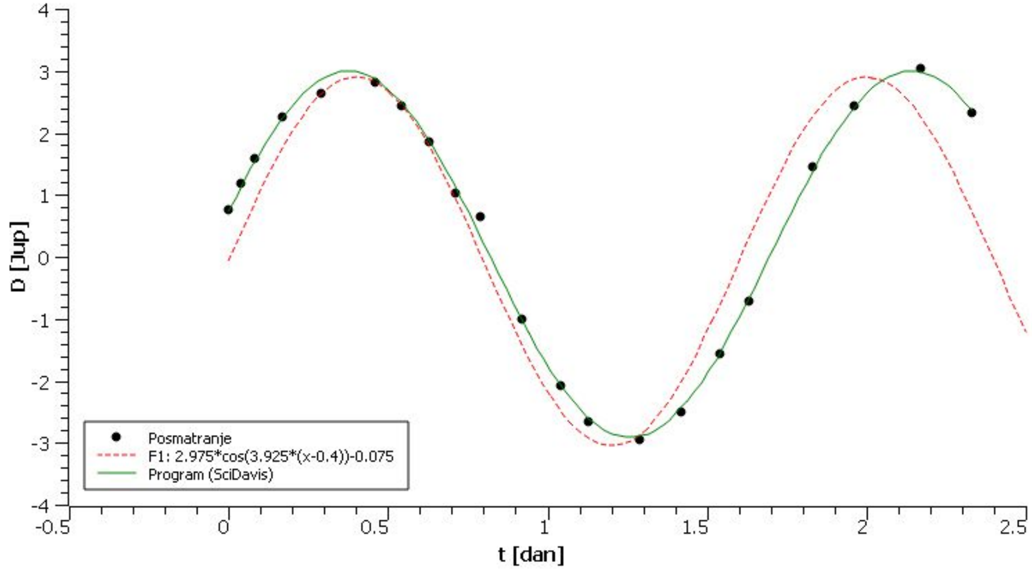
Better way (SciDavis)



https://scidavis.sourceforge.net/

$$b = -0.075,\ a = 2.975,\ \omega = 3.925,\ c = 0.4$$





Let’s compare…

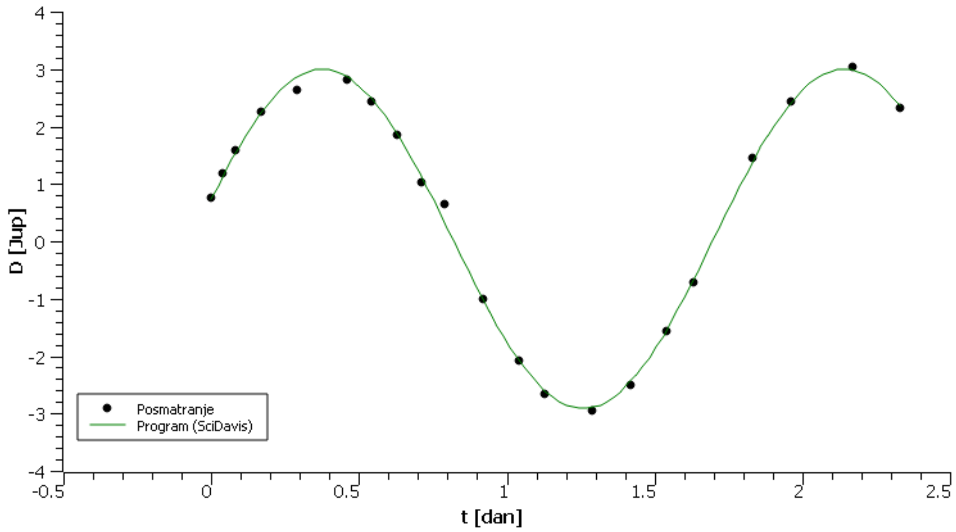
$$M = \frac{a^3}{T^2}$$

- „Paper and pen“
  - $a = 2.956$ [Jup] $= 0.00276$ AU
  - $T = 1.6$ day $= 0.0044$ year
  - $M = 0.0009134\ M_{\odot}$

$\Delta = 4.29\ \%$

- Fitting data
  - $a = 2.975$ [Jup] $= 0.00278$ AU
  - $T = 1.766$ day $= 0.0048$ year
  - $M = 0.0009311\ M_{\odot}$

$\Delta = 2.43\ \%$



$M = 0.0009543\ M_{\odot}$

„Kepler“ – space telescope

- The Kepler Space Telescope - a space observatory (launched in 2009, NASA)
  - Mission to discover Earth-like planets.
  - specifically designed to survey a portion of our region of the Milky Way to find potentially habitable exoplanets.
- Kepler's main goal was to find planets that are Earth-sized and smaller in the habitable zone, where liquid water could exist on the surface.
- Kepler used the transit method to detect exoplanets. It observed the slight dimming of a star when a planet passes in front of it, blocking a small portion of the star's light.
- Kepler discovered 2773 confirmed exoplanets and 1982 additional planet candidates.
- The mission officially ended in 2018 when Kepler ran out of fuel. It was left in a safe orbit away from Earth, and its data continues to be analyzed by scientists.
- Kepler significantly expanded the catalog of known exoplanets and showed that planets are common in our galaxy, many of which may be potentially habitable.

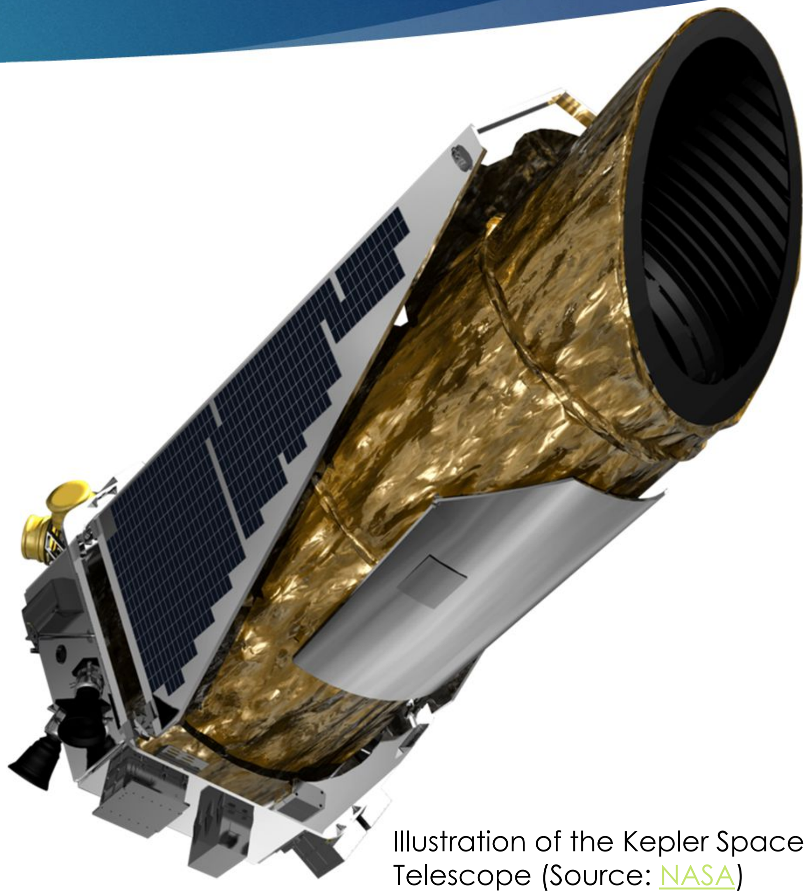
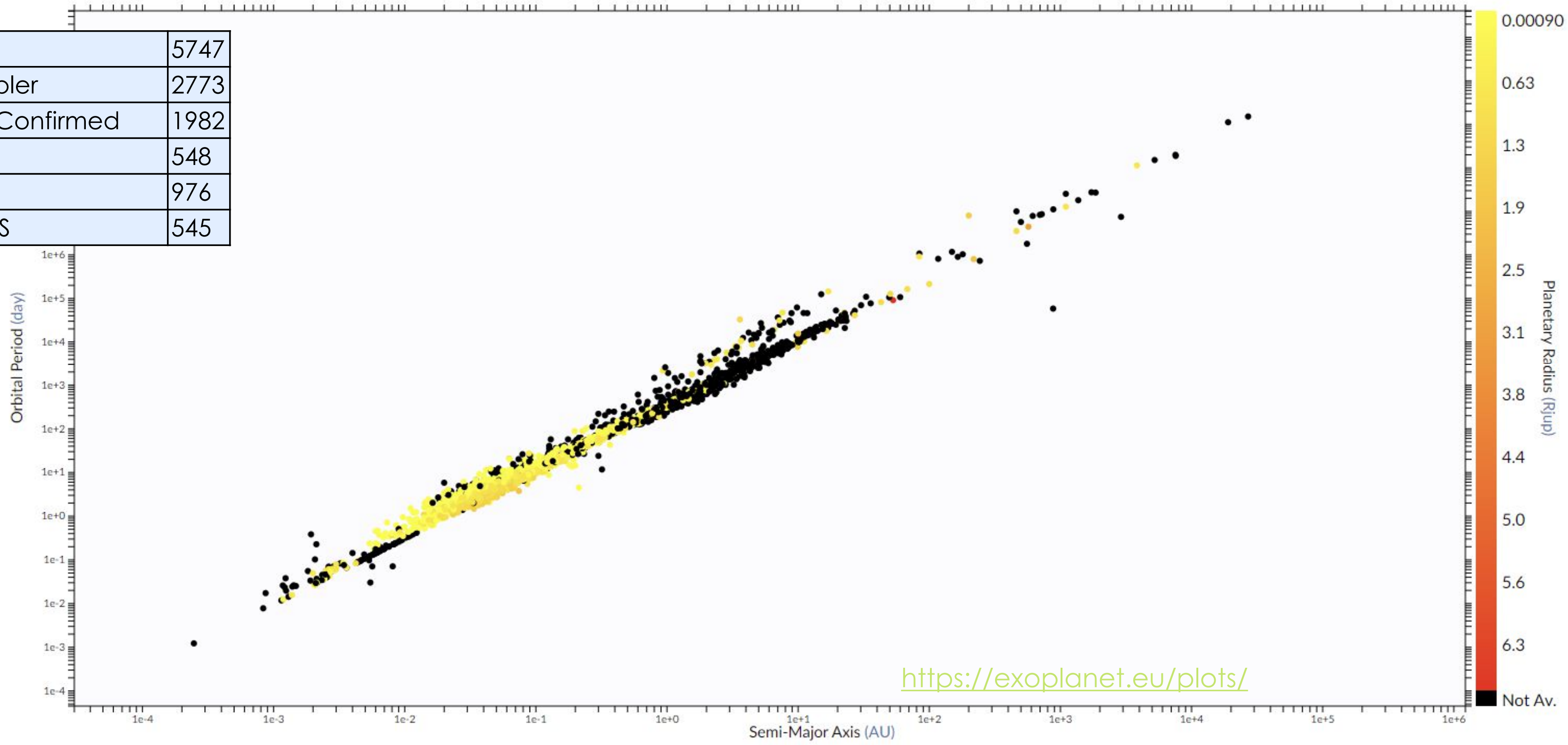


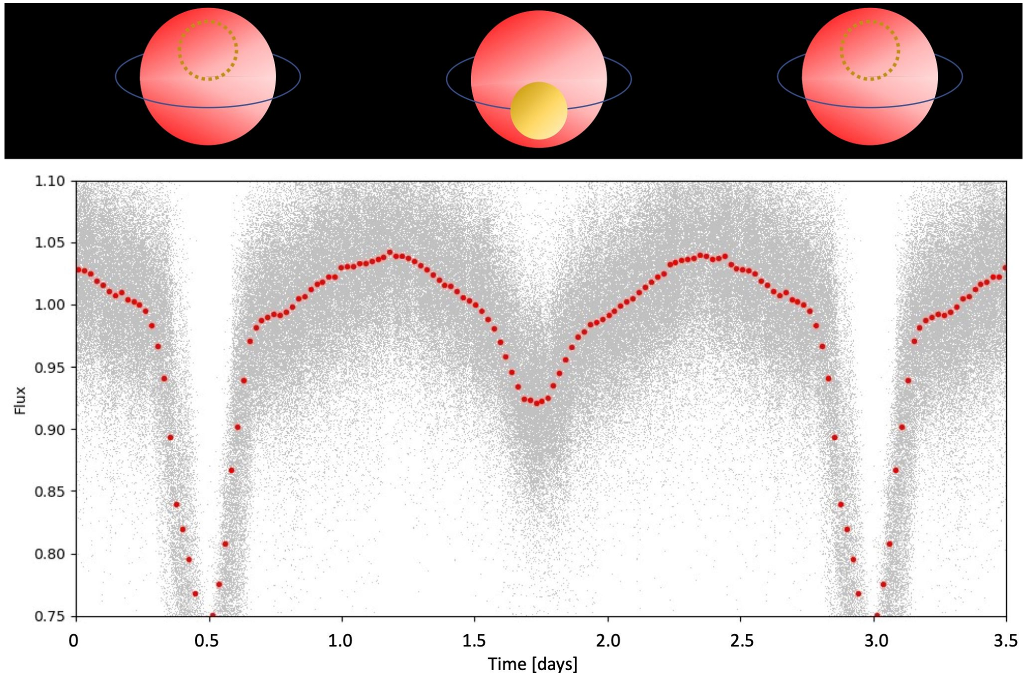
Illustration of the Kepler Space Telescope (Source: NASA)

„Kepler“ vs Kepler’s 3rd law

All Exoplanets	5747
Confirmed Planets Discovered by Kepler	2773
Kepler Project Candidates Yet To Be Confirmed	1982
Confirmed Planets Discovered by K2	548
K2 Candidates Yet To Be Confirmed	976
Confirmed Planets Discovered by TESS	545



Detection of exoplanets

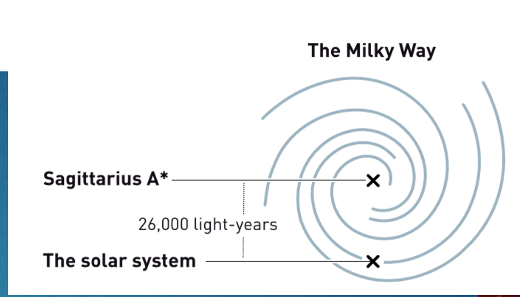


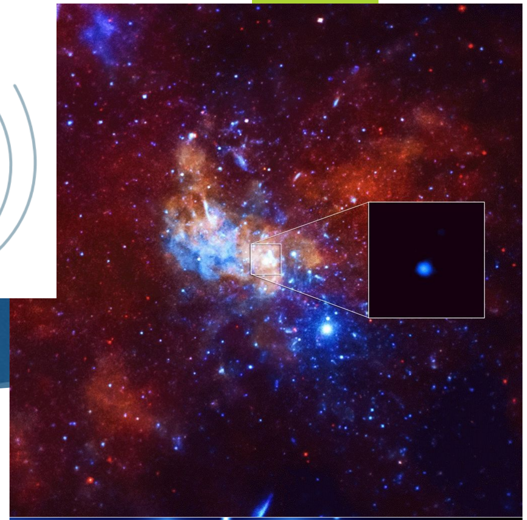
Main sequence stars (V)

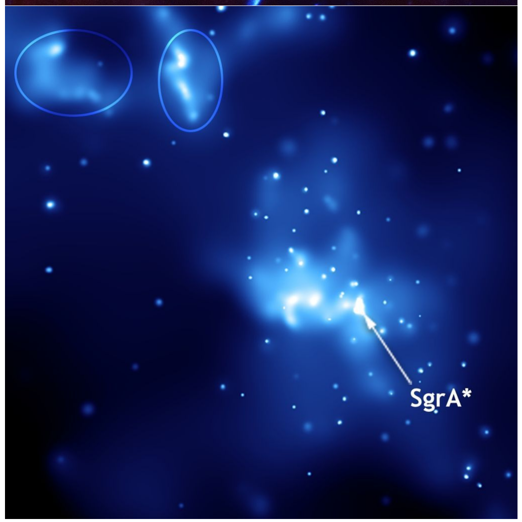
Spectral Type	Temperature (K)	Absolute Magnitude	Luminosity (in solar luminosities)	Mass (in solar masses)
O5	54,000	-10.0	846,000	30.3
O6	45,000	-8.8	275,000	22.9
O7	43,300	-8.6	220,000	21.7
O8	40,600	-8.2	150,000	19.7
O9	37,800	-7.7	95,000	17.6
B0	29,200	-6.0	20,000	12.0
B1	23,000	-4.4	4600	8.24
B2	21,000	-3.8	2600	7.14
B3	17,600	-2.6	900	5.48
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

$$M = \frac{a^3}{T^2}$$

„Our“ SMBH, *Sagitarius A**



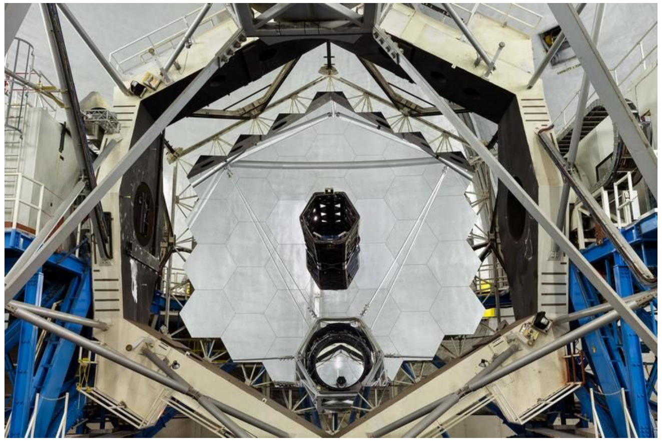




- Strong and compact radio source at the center of the Galaxy
  - Near the border of the Sagittarius and Scorpius constellations
- Since the discovery of quasars – the hypothesis of an SMBH (Supermassive Black Hole) at the center of large galaxies
  - Mass ranging from a few million to several billion solar masses
- Galaxy center Harlow Shapley (100 years ago)
  - Later shown to be Sagittarius A*
- 1990s and beyond
  - Projects by **R. Genzel and A. Ghez** – observing the orbits of stars in the center of the Milky Way

Telescopes

- R. Genzel and group
  - New Technology Telescope (La Silla mountain, Chile)
  - Very Large Telescope facility, VLT (Chile)4 telescopes, largest is 8 meters (2 times larger than NTT)
- A. Ghez and group
  - Keck Observatory (Hawaii)
  - About 10 meters (36 hexagonal segments)

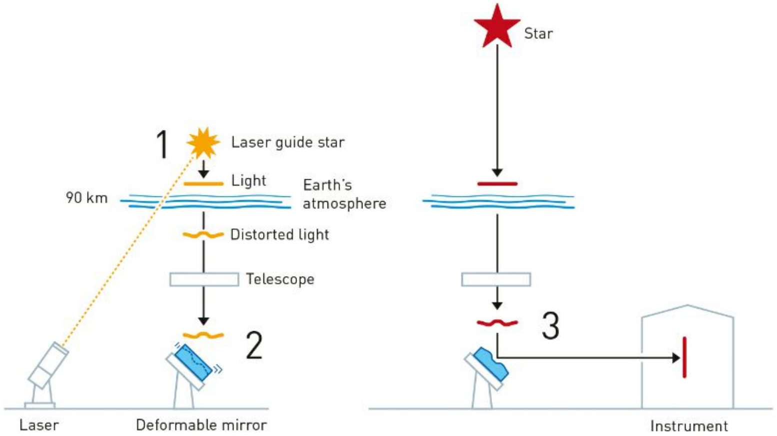


Images:

1. ESO/G. Hüdepohl (atacamaphoto.com)
2. Andrew Richard Hara/W. M. Keck Observatory [link]
3. Keck Observatory

The stars tell a story

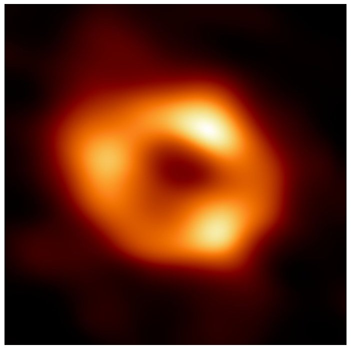
- Huge telescopes but a problem – the atmosphere
  - Adaptive optics
- Researchers tracked around 30 bright stars
  - 1 light month around the center
    - High star velocities
  - Greater distances – more stable and "standard" orbits
- **Star S2**
  - Period of 16 years – the entire orbit has been mapped!
  - (The Sun: 200 million years)

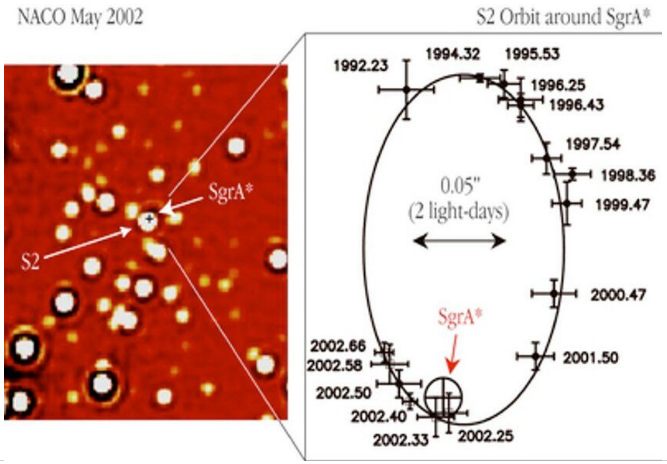


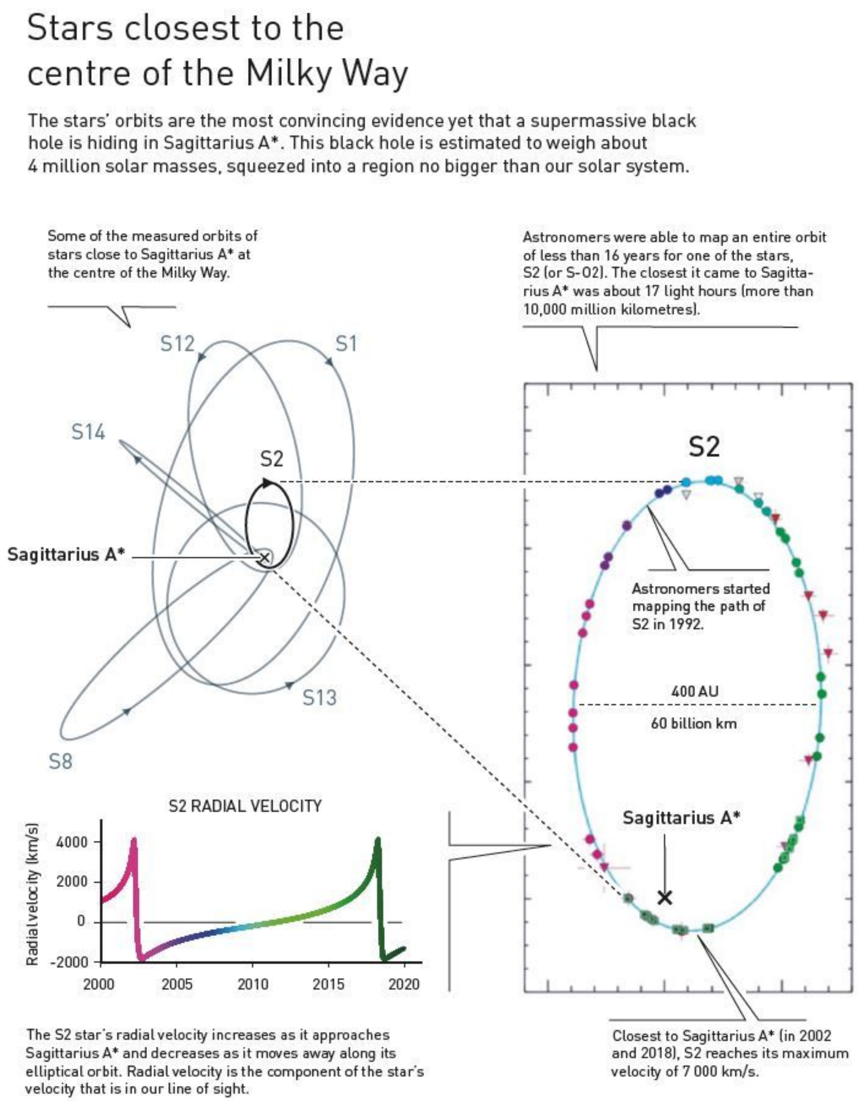
Scientific Background on the Nobel Prize in Physics 2020

The stars tell a story

- Excellent agreement in the results of both teams
  - SMBH with 4 million solar masses
  - An area the size of the Solar System
- Maybe ~~we'll "see"~~ it soon...
  - We have seen it 😊

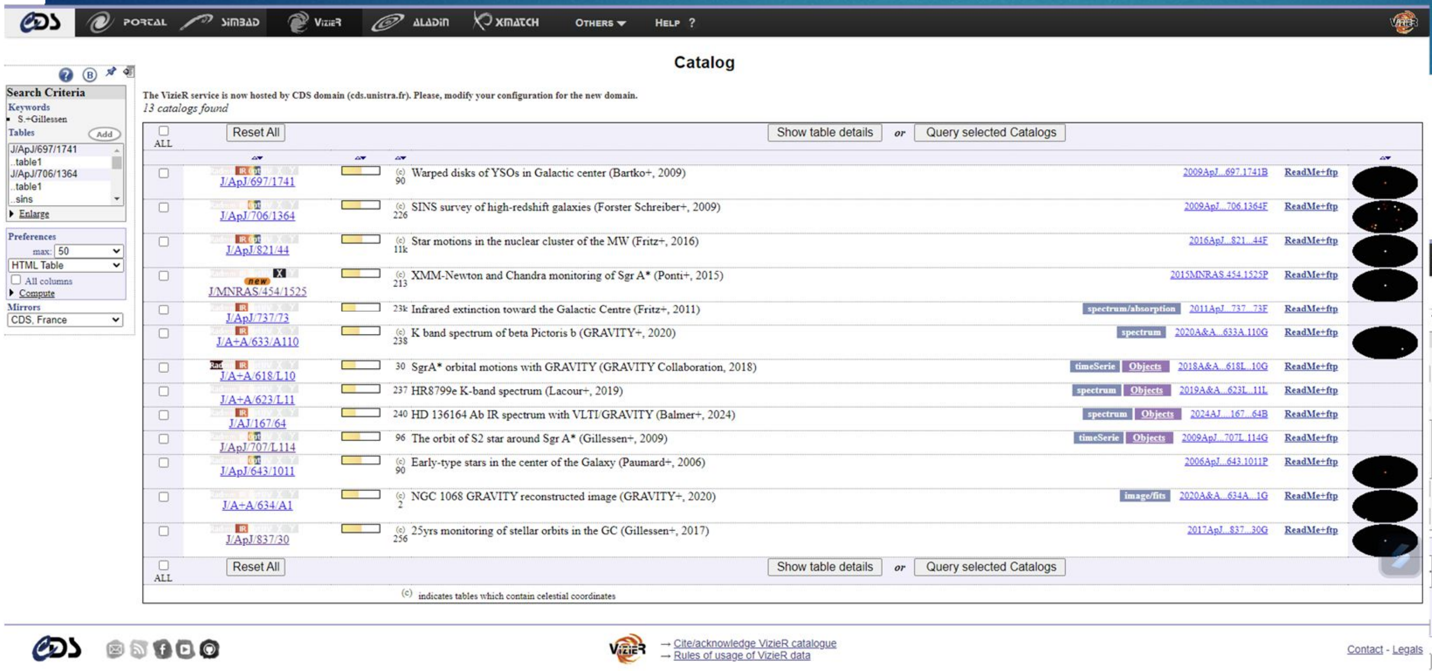


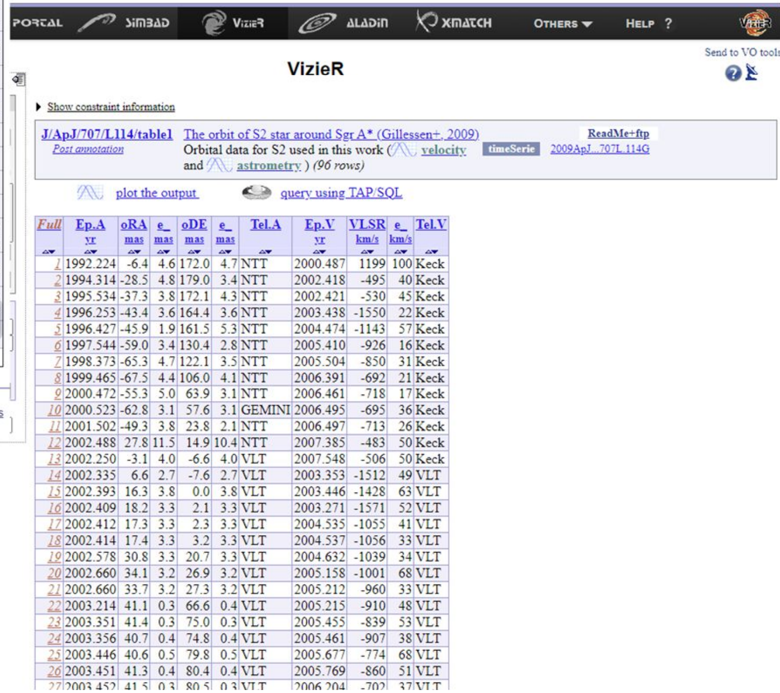




https://www.eso.org/public/images/eso0226c/

Calculate the mass of SMBH





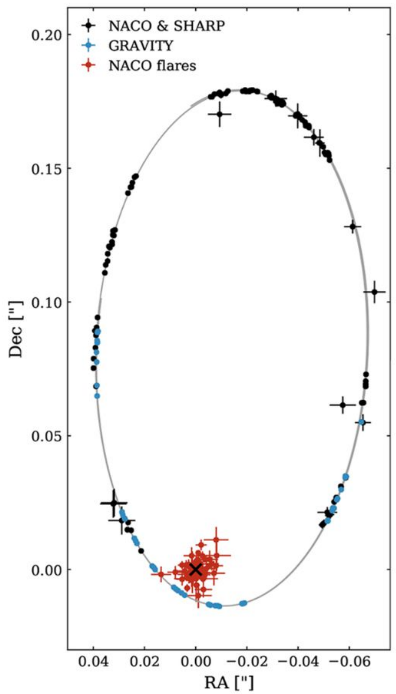
Full	Ep.A yr	oRA mas	e_ mas	oDE mas	e_ mas	Tel.A	Ep.V yr	VLSR km/s	e_ km/s	Tel.V
1	1992.224	-6.4	4.6	172.0	4.7	NTT	2000.487	1199	100	Keck
2	1994.314	-28.5	4.8	179.0	3.4	NTT	2002.418	-495	40	Keck
3	1995.534	-37.3	3.8	172.1	4.3	NTT	2002.421	-530	45	Keck
4	1996.253	-43.4	3.6	164.4	3.6	NTT	2003.438	-1550	22	Keck
5	1996.427	-45.9	1.9	161.5	5.3	NTT	2004.474	-1143	57	Keck
6	1997.544	-59.0	3.4	130.4	2.8	NTT	2005.410	-926	16	Keck
7	1998.373	-65.3	4.7	122.1	3.5	NTT	2005.504	-850	31	Keck
8	1999.465	-67.5	4.4	106.0	4.1	NTT	2006.391	-692	21	Keck
9	2000.472	-55.3	5.0	63.9	3.1	NTT	2006.461	-718	17	Keck
10	2000.523	-62.8	3.1	57.6	3.1	GEMINI	2006.495	-695	36	Keck
11	2001.502	-49.3	3.8	23.8	2.1	NTT	2006.497	-713	26	Keck
12	2002.488	27.8	11.5	14.9	10.4	NTT	2007.385	-483	50	Keck
13	2002.250	-3.1	4.0	-6.6	4.0	VLT	2007.548	-506	50	Keck
14	2002.335	6.6	2.7	-7.6	2.7	VLT	2003.353	-1512	49	VLT
15	2002.393	16.3	3.8	0.0	3.8	VLT	2003.446	-1428	63	VLT
16	2002.409	18.2	3.3	2.1	3.3	VLT	2003.271	-1571	52	VLT
17	2002.412	17.3	3.3	2.3	3.3	VLT	2004.535	-1055	41	VLT
18	2002.414	17.4	3.3	3.2	3.3	VLT	2004.537	-1056	33	VLT
19	2002.578	30.8	3.3	20.7	3.3	VLT	2004.632	-1039	34	VLT
20	2002.660	34.1	3.2	26.9	3.2	VLT	2005.158	-1001	68	VLT
21	2002.660	33.7	3.2	27.3	3.2	VLT	2005.212	-960	33	VLT
22	2003.214	41.1	0.3	66.6	0.4	VLT	2005.215	-910	48	VLT
23	2003.351	41.4	0.3	75.0	0.3	VLT	2005.455	-839	53	VLT
24	2003.356	40.7	0.4	74.8	0.4	VLT	2005.461	-907	38	VLT
25	2003.446	40.6	0.5	79.8	0.5	VLT	2005.677	-774	68	VLT
26	2003.451	41.3	0.4	80.4	0.4	VLT	2005.769	-860	51	VLT
27	2003.452	41.5	0.3	80.5	0.3	VLT	2006.204	-702	37	VLT

https://vizier.cds.unistra.fr/viz-bin/VizieR

Calculate the mass of SMBH

Source: The ESA/ESO Astronomy Exercise Series 6

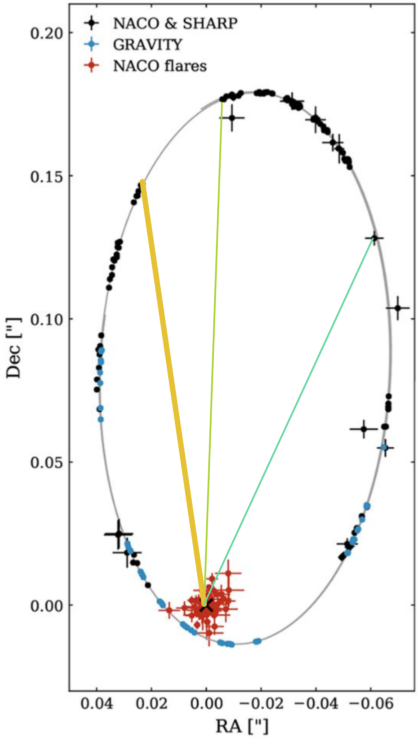
- Right – coordinates of the position of S2 star
  - Origin of the coordinate system is at the center of SMBH
- **How?**
  - Plot points (and errors bars)
  - Draw the ellipse that best fits the measurements
  - Measure the semi-major axis of the ellipse
  - Convert arcseconds to light-days (ld), 2 arcsec = 28 ld
  - Calculate the average value 😊



Date (year)	x (arcsec)	dx (arcsec)	y (arcsec)	dy (arcsec)
1992.226	0.104	0.003	-0.166	0.004
1994.321	0.097	0.003	-0.189	0.004
1995.531	0.087	0.002	-0.192	0.003
1996.256	0.075	0.007	-0.197	0.010
1996.428	0.077	0.002	-0.193	0.003
1997.543	0.052	0.004	-0.183	0.006
1998.365	0.036	0.001	-0.167	0.002
1999.465	0.022	0.004	-0.156	0.006
2000.474	-0.000	0.002	-0.103	0.003
2000.523	-0.013	0.003	-0.113	0.004
2001.502	-0.026	0.002	-0.068	0.003
2002.252	-0.013	0.005	0.003	0.007
2002.334	-0.007	0.003	0.016	0.004
2002.408	0.009	0.003	0.023	0.005
2002.575	0.032	0.002	0.016	0.003
2002.650	0.037	0.002	0.009	0.003
2003.214	0.072	0.001	-0.024	0.002
2003.353	0.077	0.002	-0.030	0.002
2003.454	0.081	0.002	-0.036	0.002

Calculate the mass of SMBH

- …
- Calculate the period (T)
  - $A_{ell} = ab \cdot \pi$ - a and b from the image (previous slide)
  - $\Delta A = \frac{\Delta t}{P} \cdot A_{ell}$ - The 2nd Kelper's law
  - Unknown ΔA, Δt, A_{ell}
    - ΔA and Δt – from image (previous slide, for each segment)
    - Plot a triangle and calculate its area (repeat this step several times!)
  - Calculate the mass of the SMBH
    - The 3rd Kepler's law
    - $T^2 = \frac{4\pi^2}{G(M+m_{S2})} a^3$, $M \gg m_{S2}$



Date (year)	x (arcsec)	dx (arcsec)	y (arcsec)	dy (arcsec)
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1997.543	0.052	0.004	-0.183	0.006
1998.365	0.036	0.001	-0.167	0.002
1999.465	0.022	0.004	-0.156	0.006
2000.474	-0.000	0.002	-0.103	0.003
2000.523	-0.013	0.003	-0.113	0.004
2001.502	-0.026	0.002	-0.068	0.003
2002.252	-0.013	0.005	0.003	0.007
2002.334	-0.007	0.003	0.016	0.004
2002.408	0.009	0.003	0.023	0.005
2002.575	0.032	0.002	0.016	0.003
2002.650	0.037	0.002	0.009	0.003
2003.214	0.072	0.001	-0.024	0.002
2003.353	0.077	0.002	-0.030	0.002
2003.454	0.081	0.002	-0.036	0.002

ChatGPT 3.5 vs Students 2:0

Part	Groups	Language	Score	Std. dev
1	ChatGPT	English	94.28	4.88
1	ChatGPT	Serbian	75.03	7.90
1	Students	Serbian	71.96	21.88
2	ChatGPT	English	91.30	7.82
2	ChatGPT	Serbian	85.25	9.04
2	Students	Serbian	73.93	24.89

ChatGPT answers collected in April 2023, students answers april – december 2022.

Radenković, Lazar, and Milan Milošević. "A Comparison of AI Performance with Student Performance in Astrophysics and Astrobiology." The Physics Teacher 62, no. 5 (May 1, 2024): 374–76. https://doi.org/10.1119/5.0168896.

Thank you

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$$\varepsilon = \sqrt{1 - \frac{b^2}{a^2}}$$

$$\frac{dA}{dt} = \frac{abn}{2}$$

$$\frac{a^3}{T^2} = M$$